

# Finite Automata

## Part One

# Computability Theory

What problems can we solve with a computer?

What problems can we solve with a computer?

What kind of  
computer?



# Two Challenges

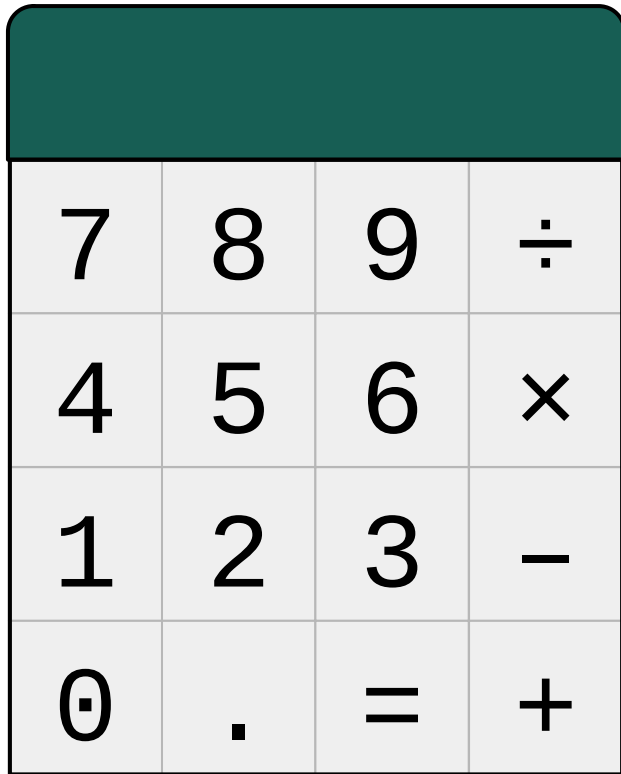
- Computers are dramatically better now than they've ever been, and that trend continues.
- Writing proofs on formal definitions is hard, and computers are *way* more complicated than sets, graphs, or functions.
- ***Key Question:*** How can we prove what computers can and can't do...
  - ... so that our results are still true in 20 years?
  - ... without multi-hundred page proofs?

# Enter Automata

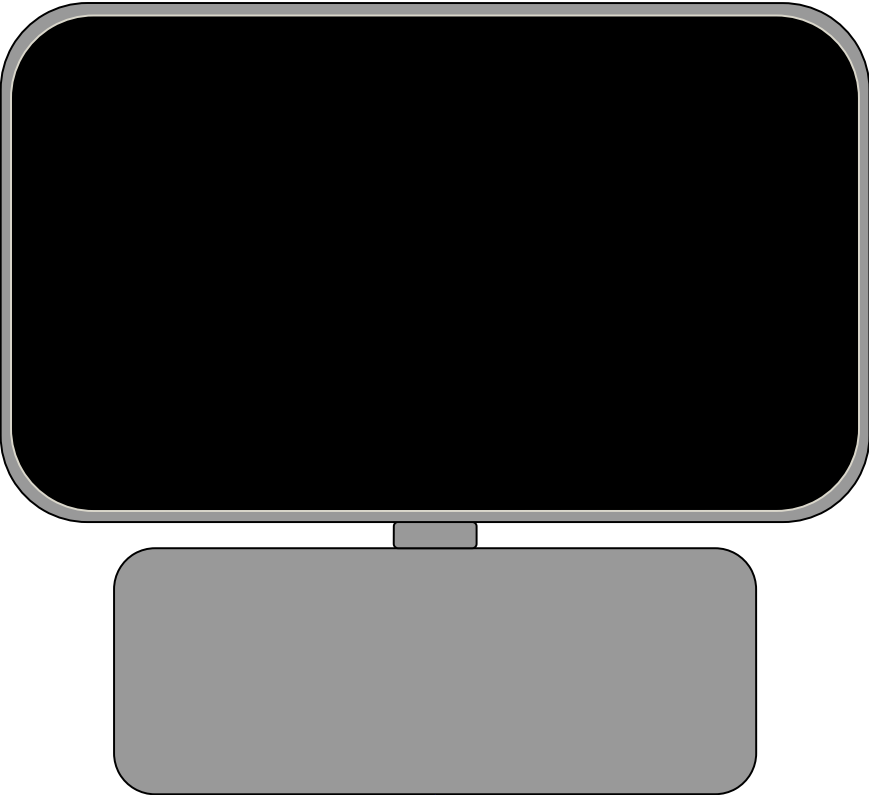
- An ***automaton*** is a mathematical model of a computing device.
- It's an ***abstraction*** of a real computer, the way that graphs are abstractions of social networks, transportation grids, etc.
- The automata we'll explore are
  - powerful enough to capture huge classes of computing devices, yet
  - simple enough that we can reason about them in a small space.

What do these automata look like?

# A Tale of Two Computers



Why does this computer...



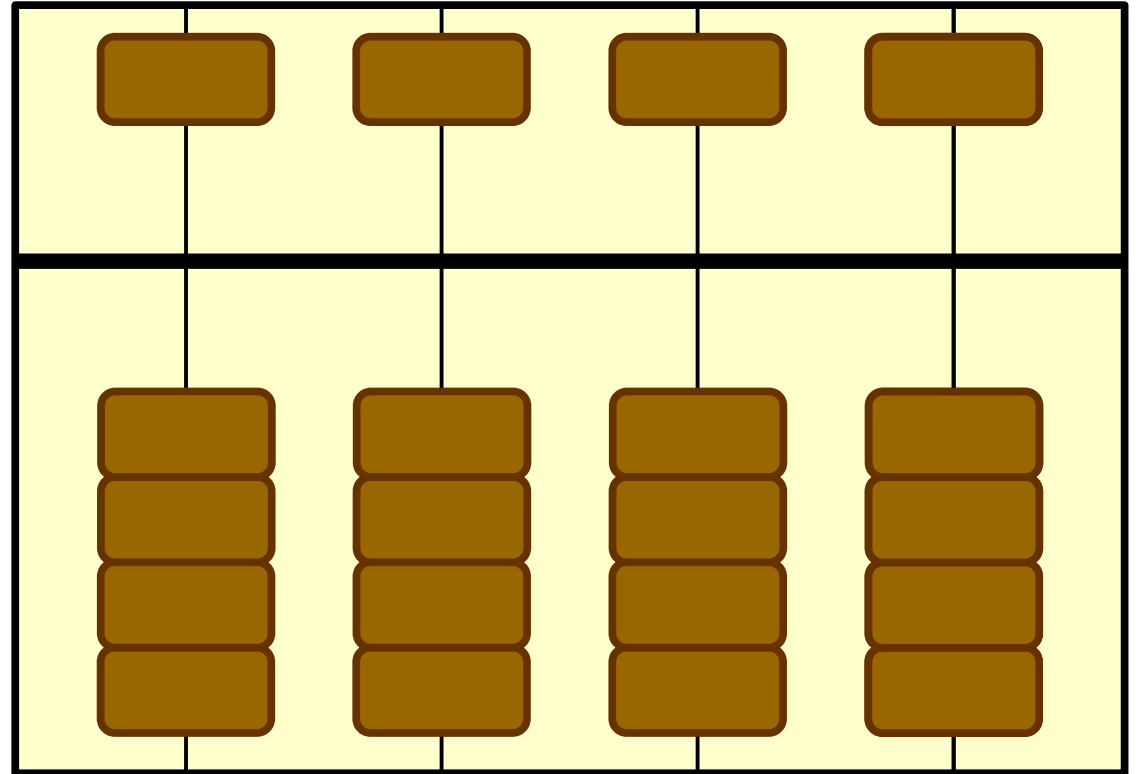
...“feel” less powerful than this one?

# Calculators vs. Desktops

- A calculator has a ***small amount of memory***. A desktop computer has a ***large amount of memory***.
- A calculator performs a ***fixed set of functions***. A desktop is ***reprogrammable*** and can run many different programs.
- These two distinctions account for much of the difference between “calculator-like” computers and “desktop-esque” computers.
- In CS103, we’ll first explore “small-memory” computers in detail, then discuss “large-memory” computers in depth.

# Computing with Finite Memory

Calculator			
7	8	9	÷
4	5	6	×
1	2	3	-
0	.	=	+



Data stored electronically. Algorithm is in silicon.  
Memory limited by display.

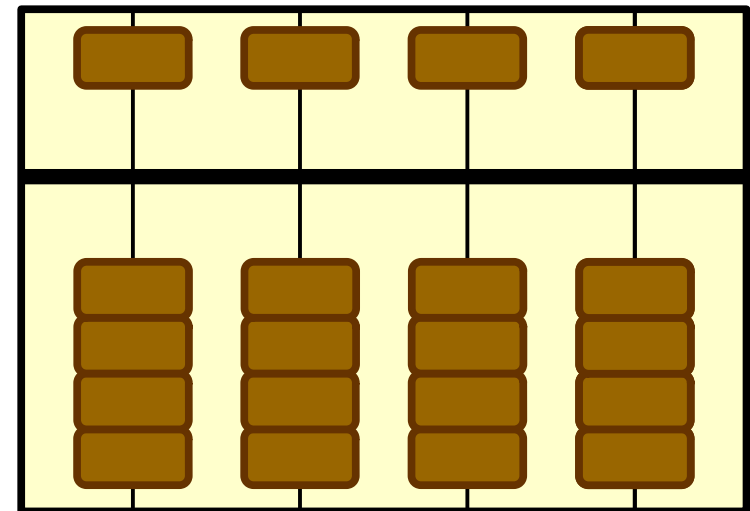
Data stored in wood. Algorithm is in brain.  
Memory limited by beads.

How do we model “memory” and  
“an algorithm” when they can take  
on so many forms?

# What's in Common?

- These machines **receive input** from an external source.
- That input is provided **sequentially**, one discrete unit at a time.
- Each input causes the device to **change configuration**. This change, big or small, is where the computation happens.

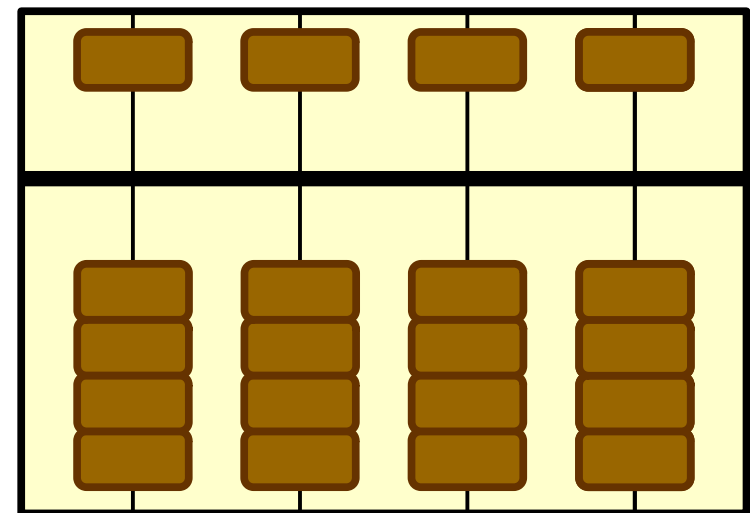
Calculator Keypad			
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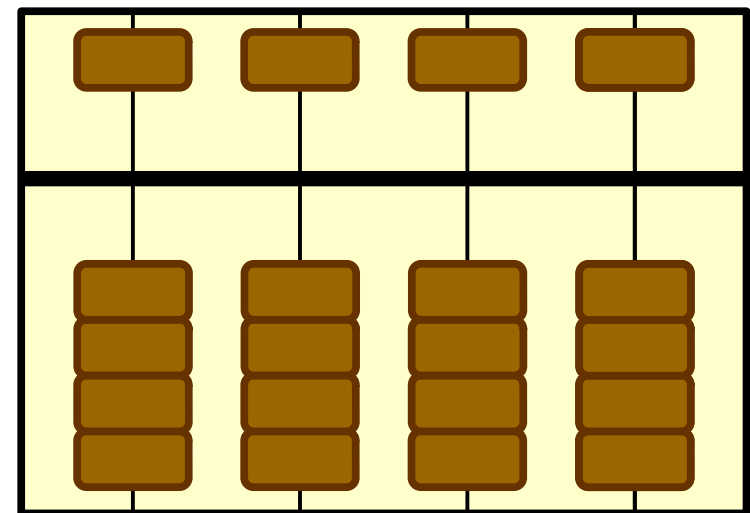
1			
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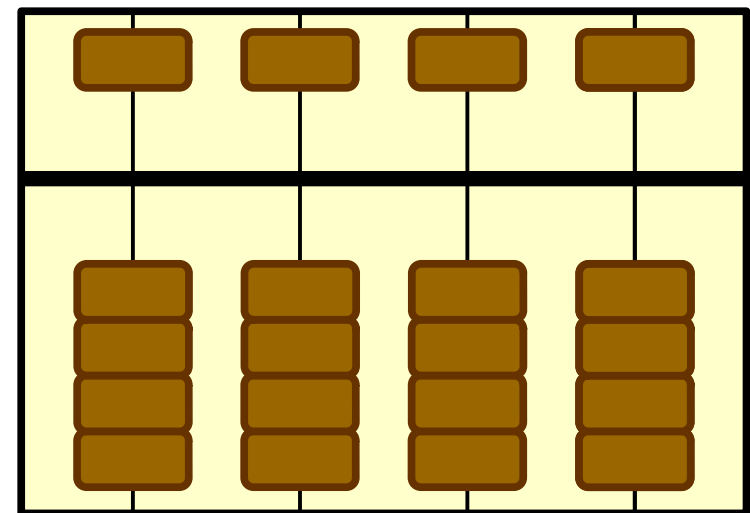
13			
7	8	9	÷
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1	2	3	-
0	.	=	+



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137			
7	8	9	÷
4	5	6	×
1	2	3	-
0	.	=	+

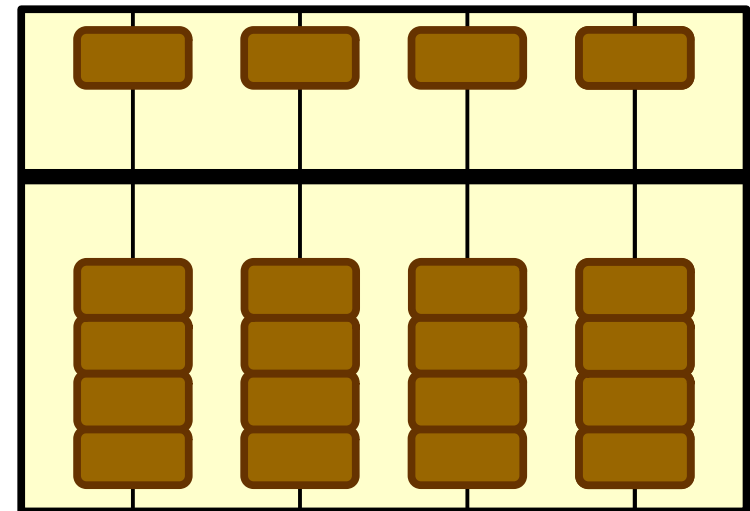


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+			
7	8	9	÷
4	5	6	×
1	2	3	-
0	.	=	+

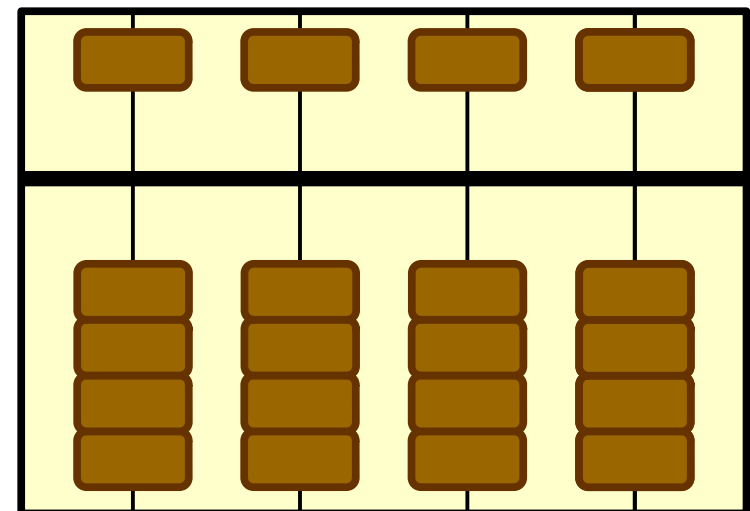
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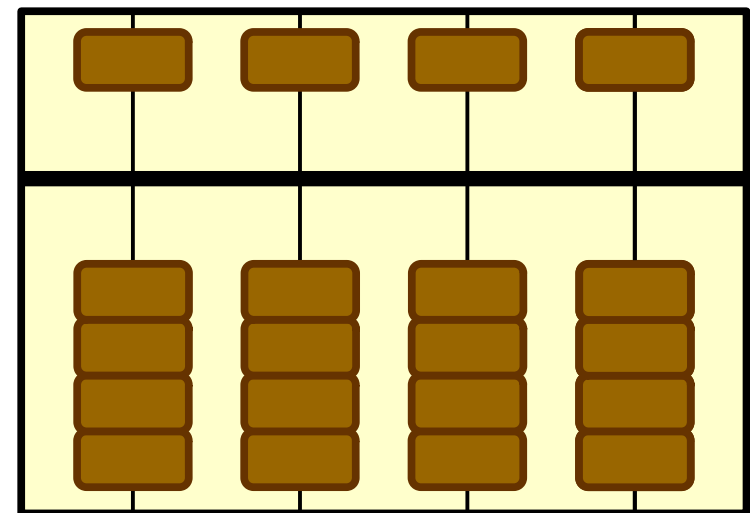
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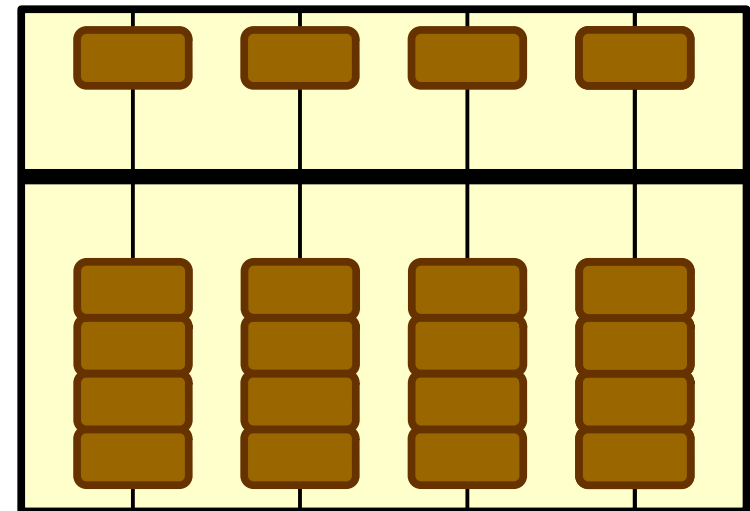
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4	5	6	×
1	2	3	-
0	.	=	+



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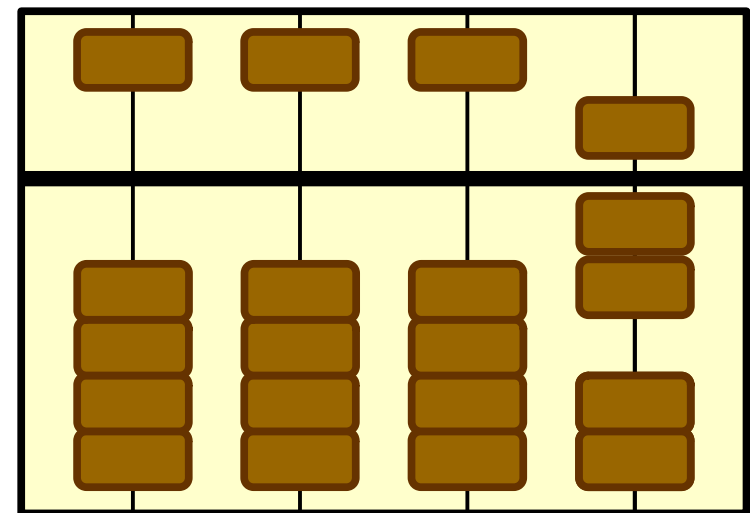
179			
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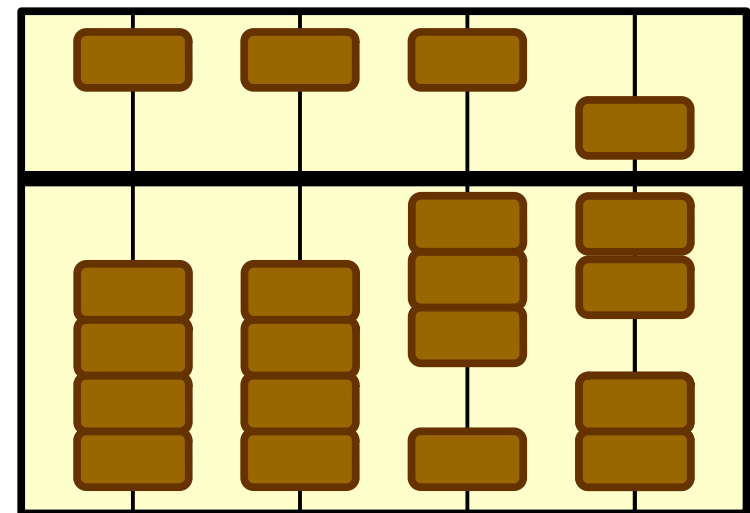
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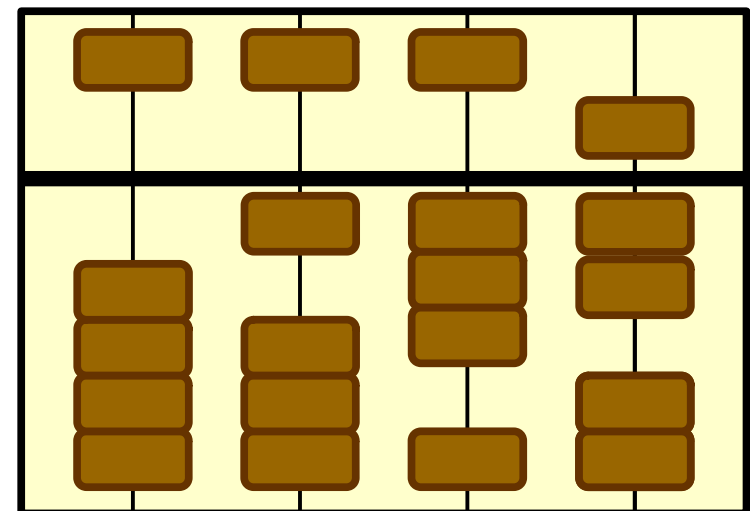
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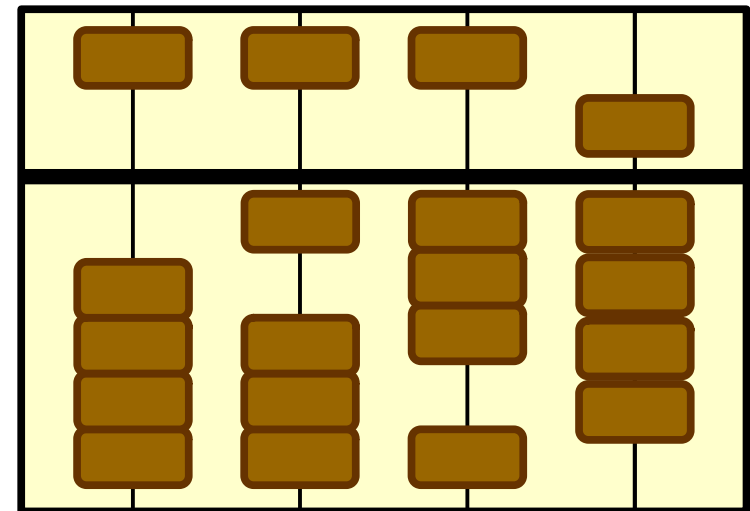
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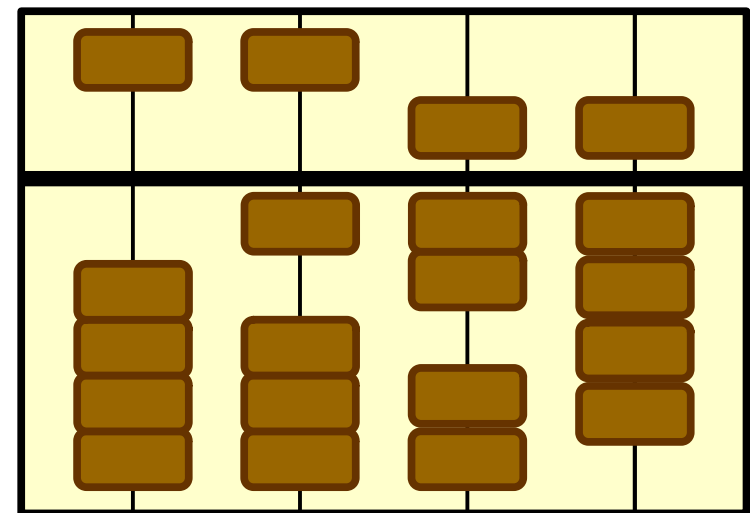
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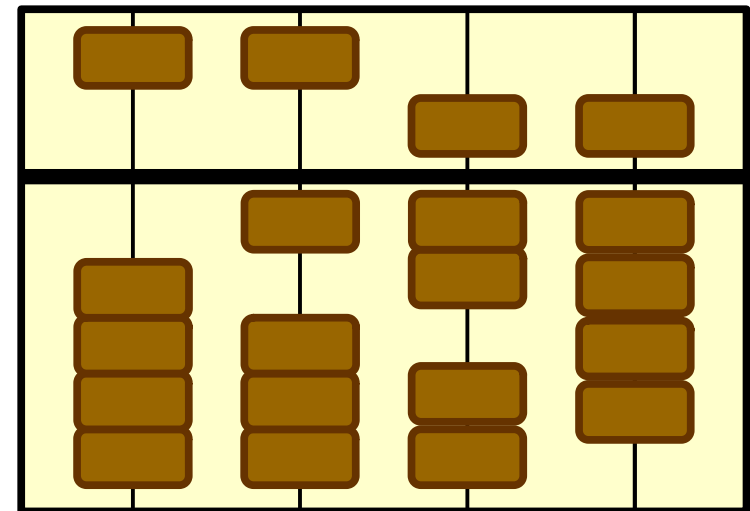
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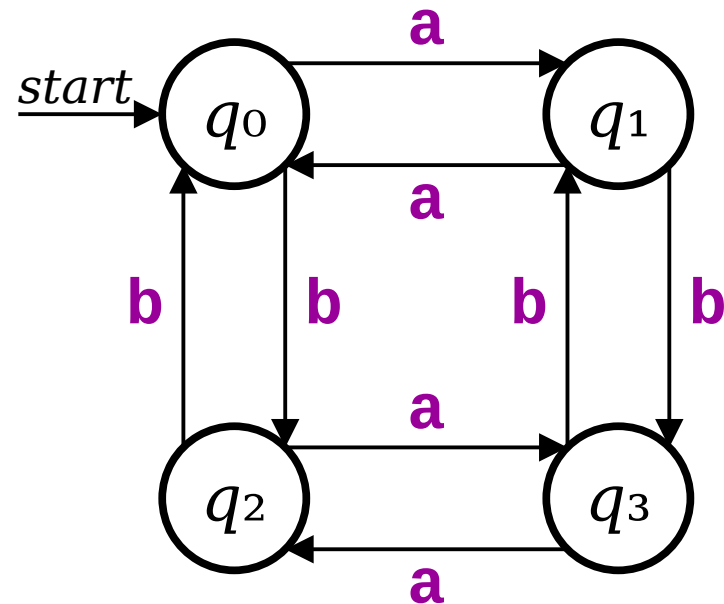
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- That input is provided **sequentially**, one discrete unit at a time.
- Each input causes the device to **change configuration**. This change, big or small, is where the computation happens.
- Once all input is provided, we can **read off an answer** based on the configuration of the device.

179			
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4	5	6	×
1	2	3	-
0	.	=	+



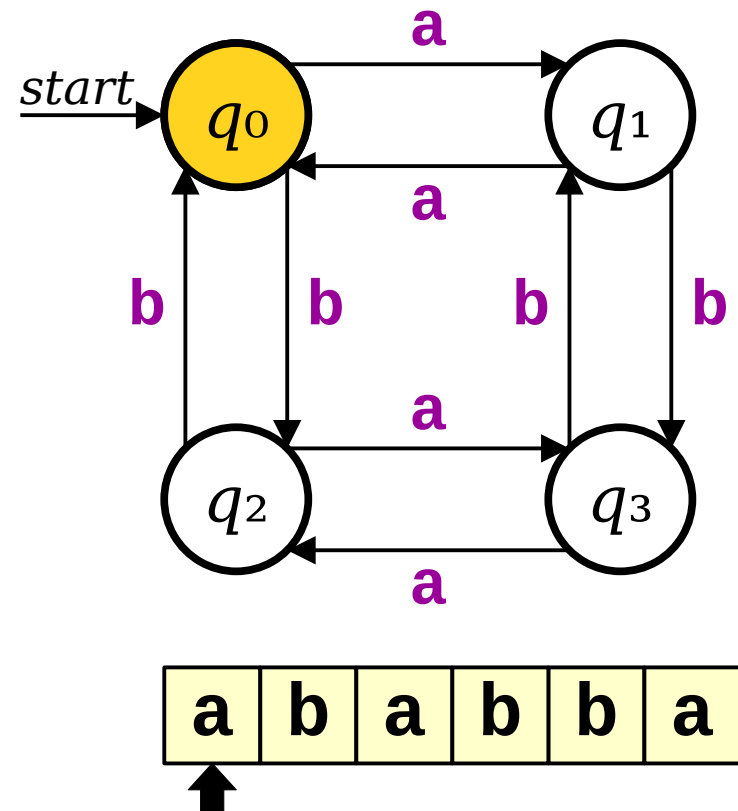
# Modeling Finite Computation

- We will model a finite-memory computer as a collection of **states** linked by **transitions**.
- Each state corresponds to one possible configuration of the device's memory.
- Each transition indicates how memory changes in response to inputs.
- Some state is designated as the **start state**. The computation begins in that state.



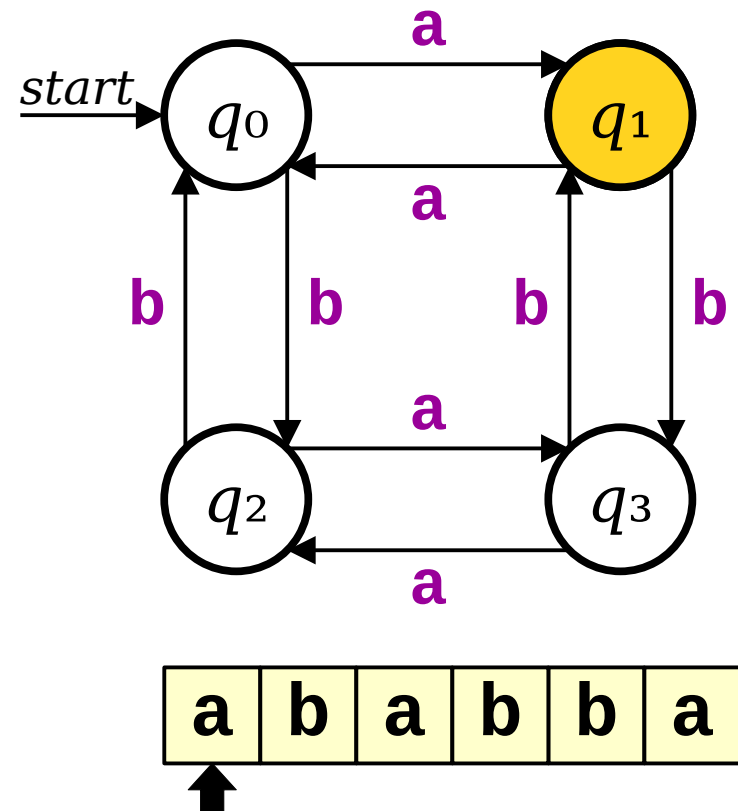
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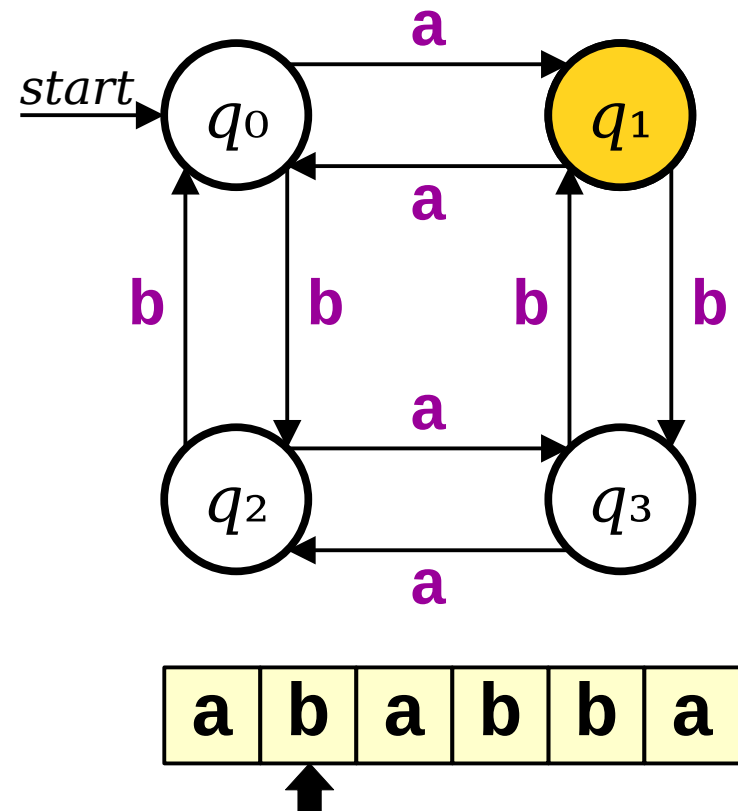
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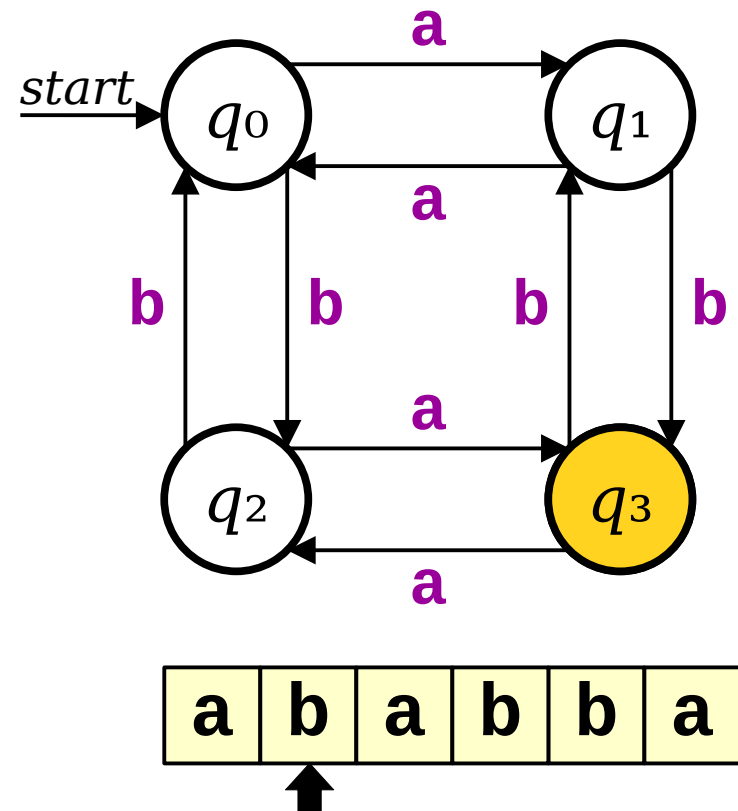
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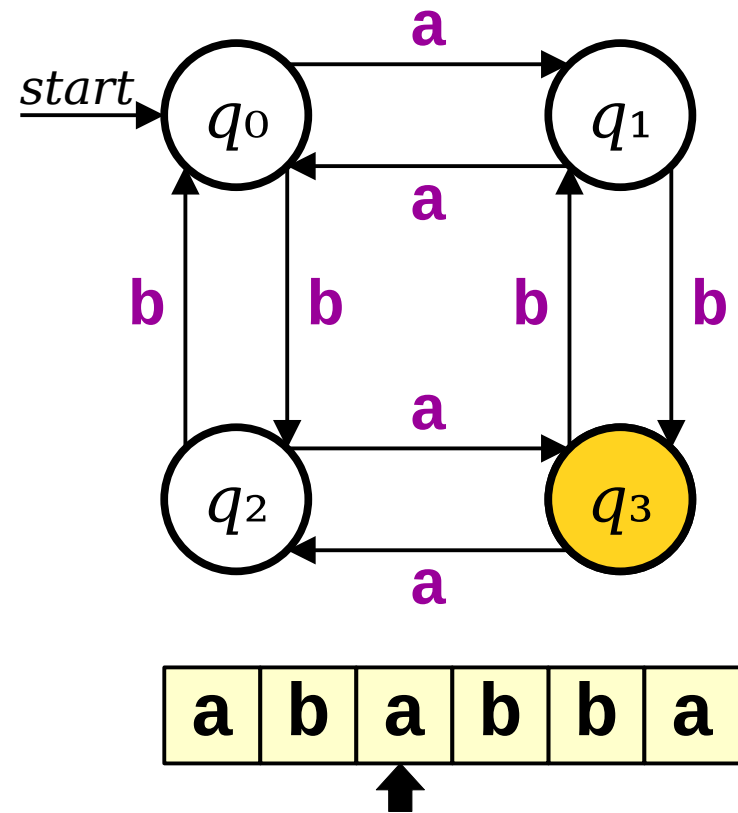
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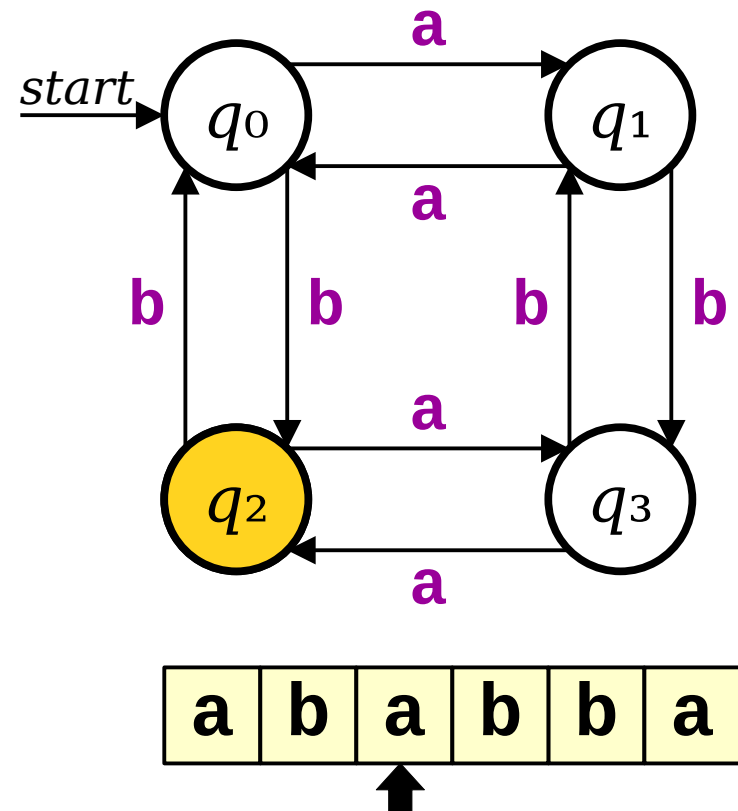
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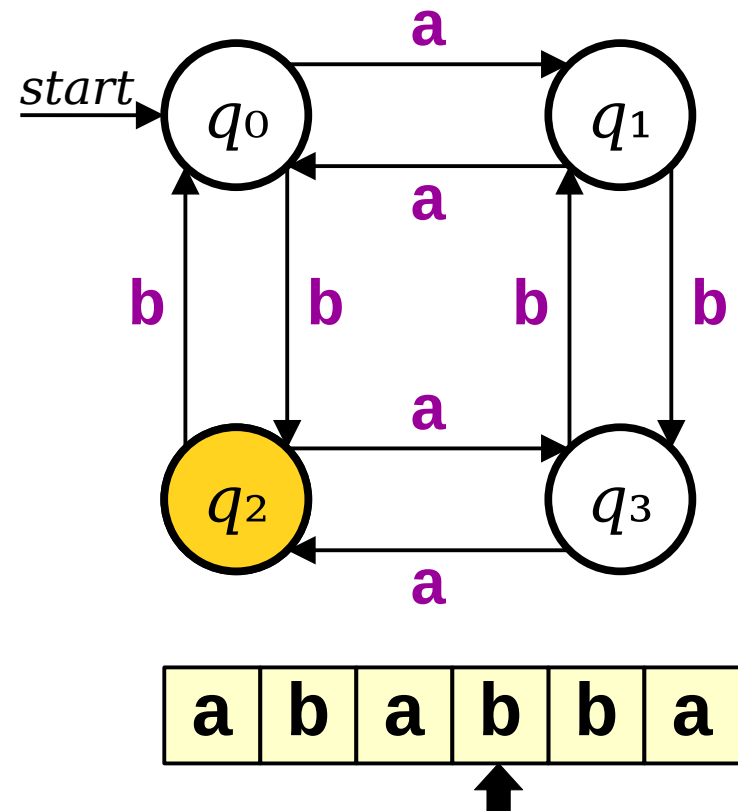
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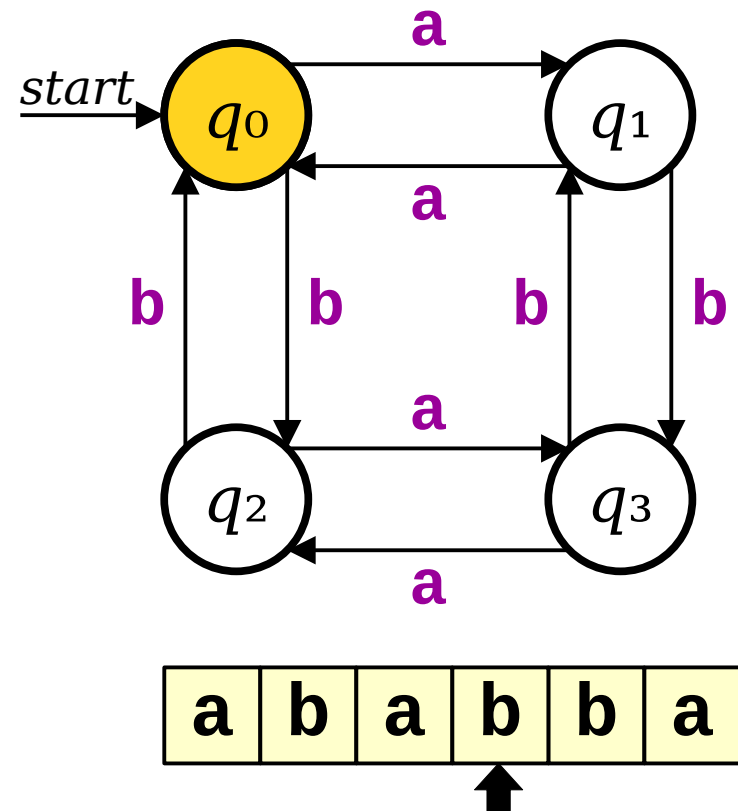
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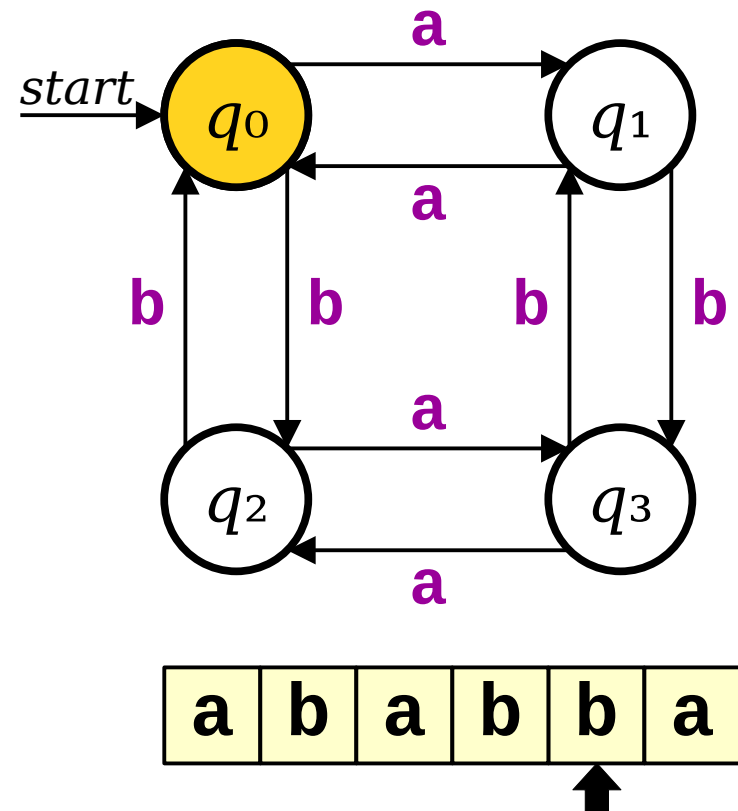
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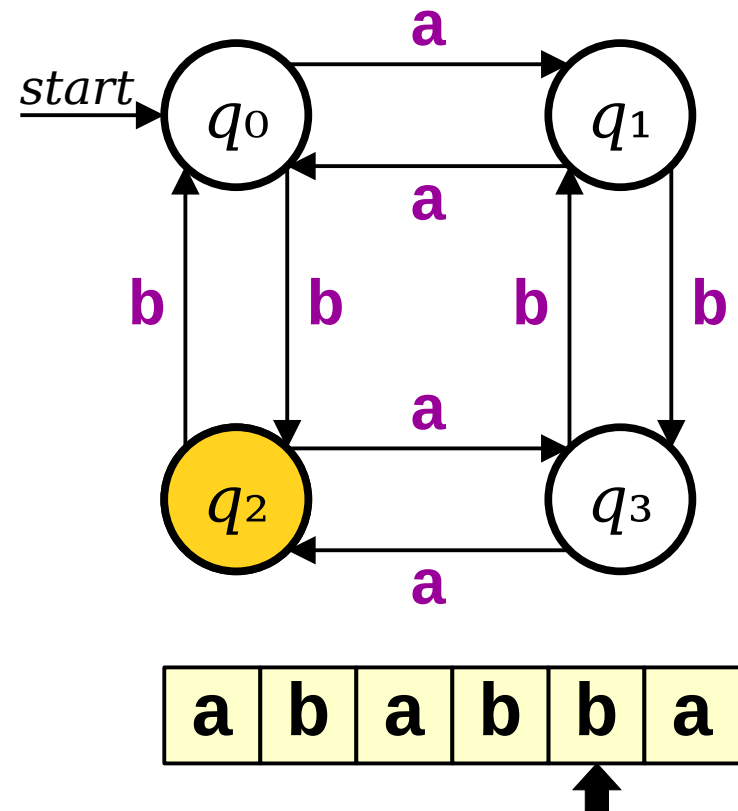
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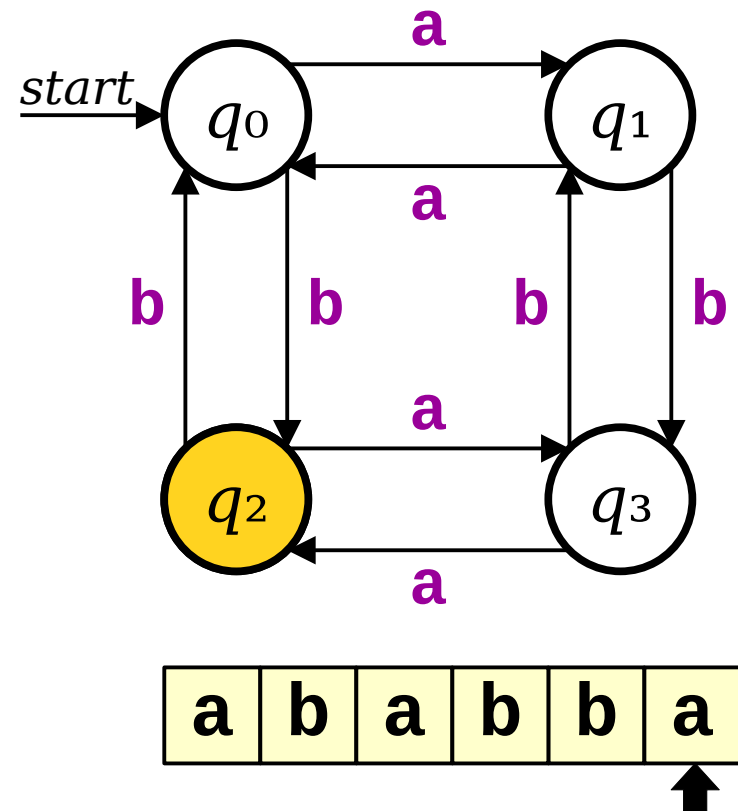
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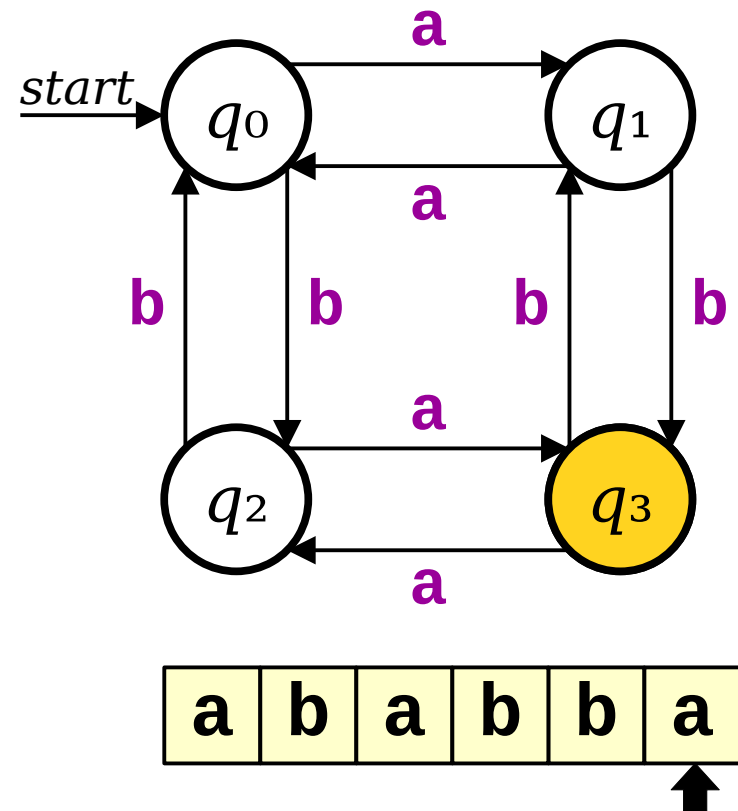
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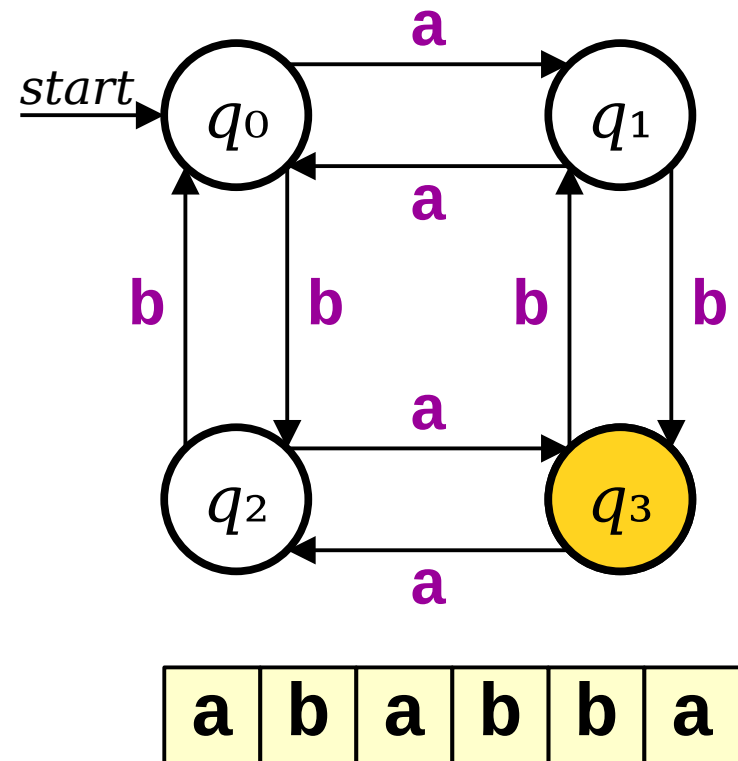
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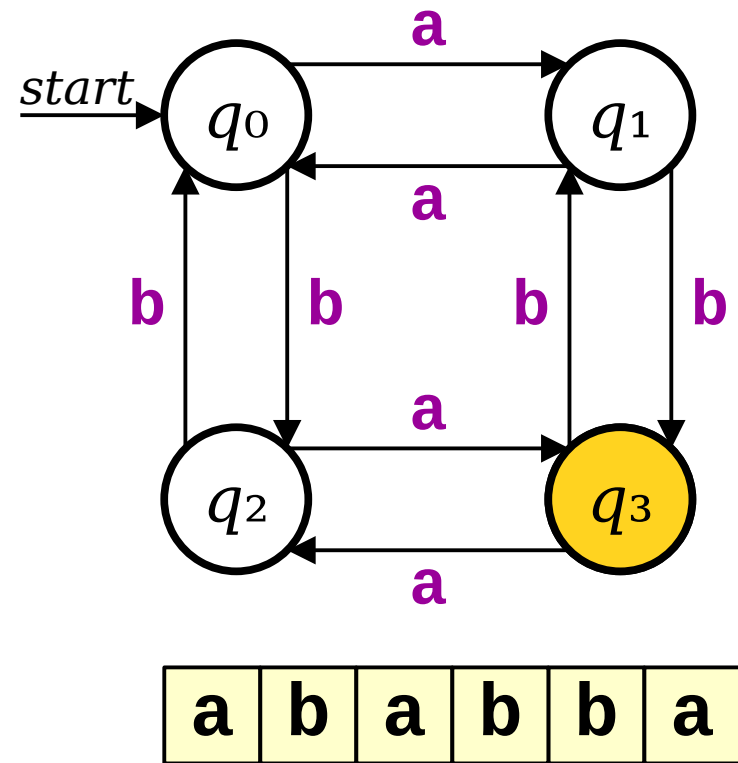
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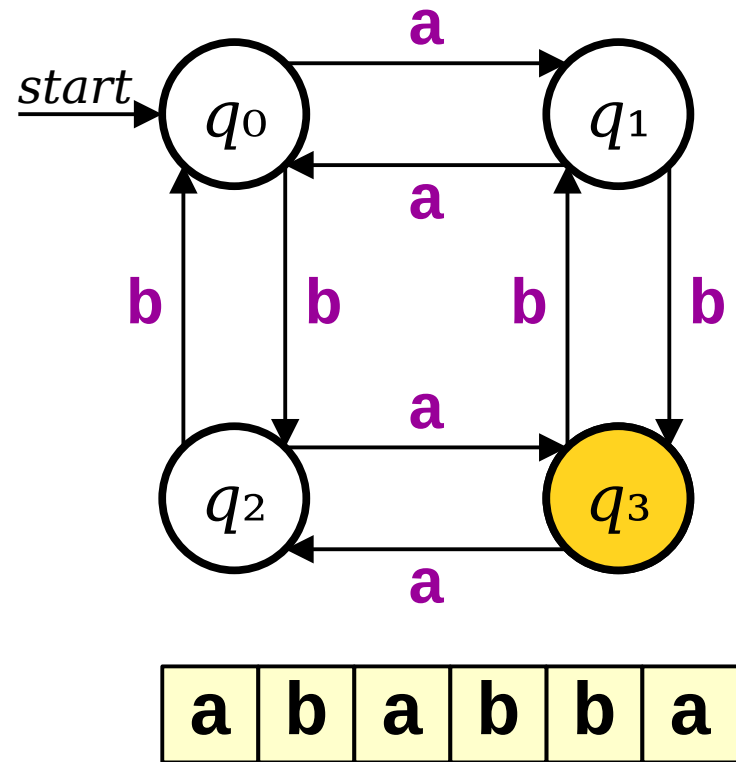
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- Once we've finished entering all the characters of our input, we need to obtain the result of the computation.
- In general, computers can produce all sorts of things as the result of a computation: a number, a piece of text, etc.
- As a simplifying assumption, we'll assume that we just need to get a single bit of output. That is, our machines will just say YES or NO.
- (This can be generalized – come talk to me after class if you're curious how!)



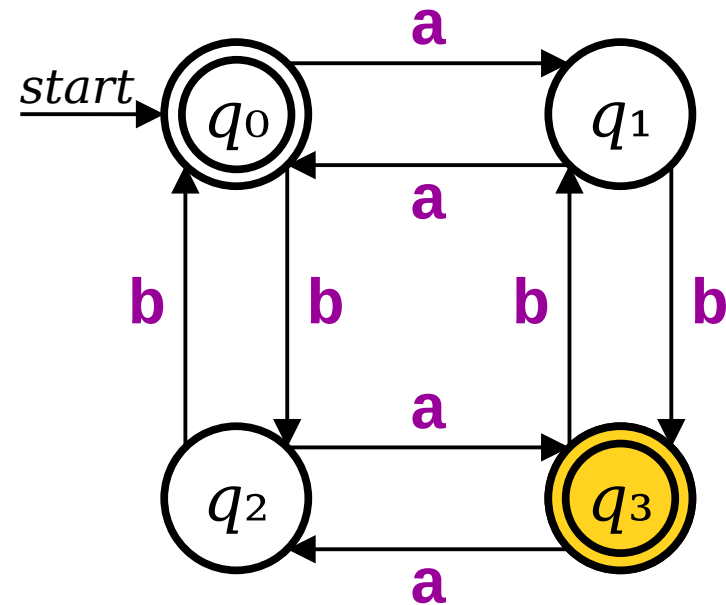
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- Some of the states in our computational device will be marked as **accepting states**. These are denoted with a double ring.



# Modeling Finite Computation

- Some of the states in our computational device will be marked as **accepting states**. These are denoted with a double ring.
- If the device ends in an accepting state after seeing all the input, **accepts** the input (says YES)
- If the device does not end in an accepting state after seeing all the input, it **rejects** the input (says NO).



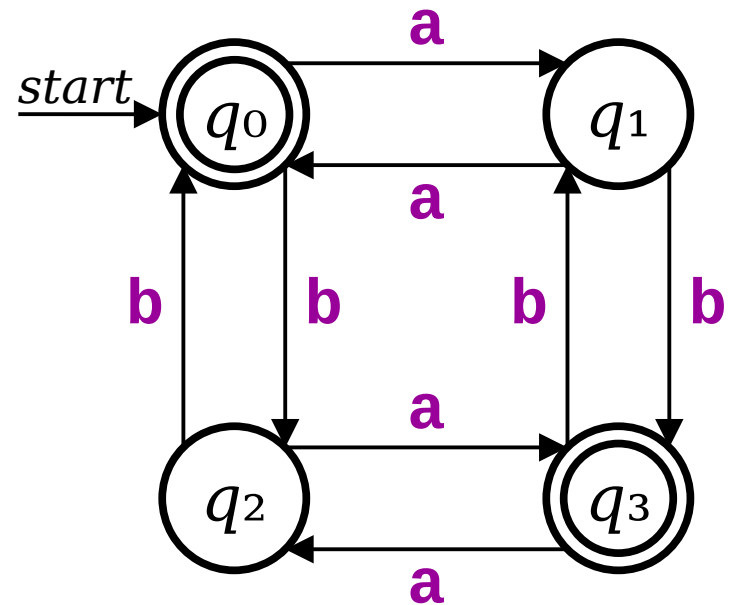
# Modeling Finite Computation

- Try it yourself!  
Which of these strings does this device accept?

**aab**

**aabb**

**abbababba**



***Respond at [pollev.com/zhenglian740](https://pollev.com/zhenglian740)***

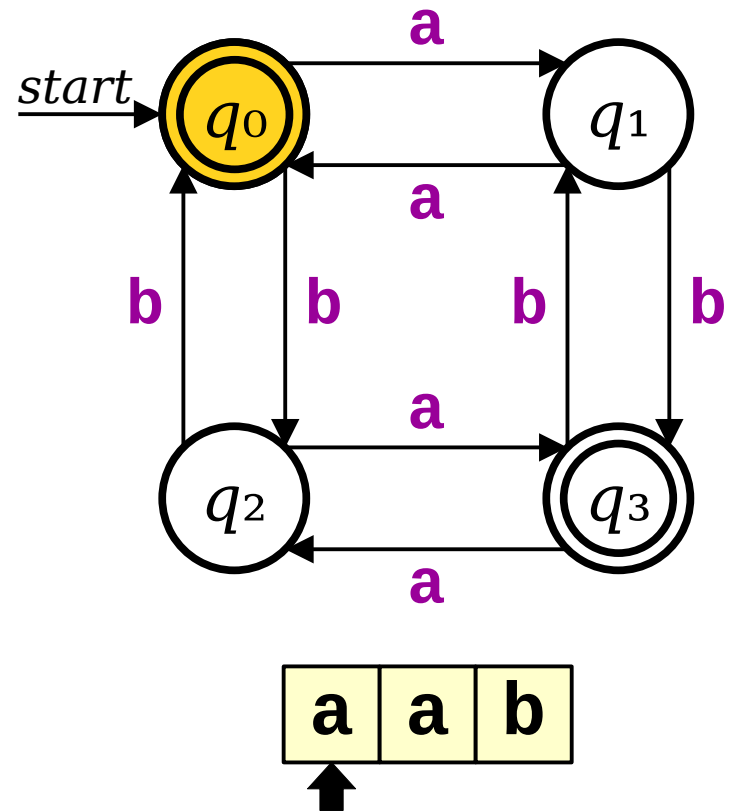
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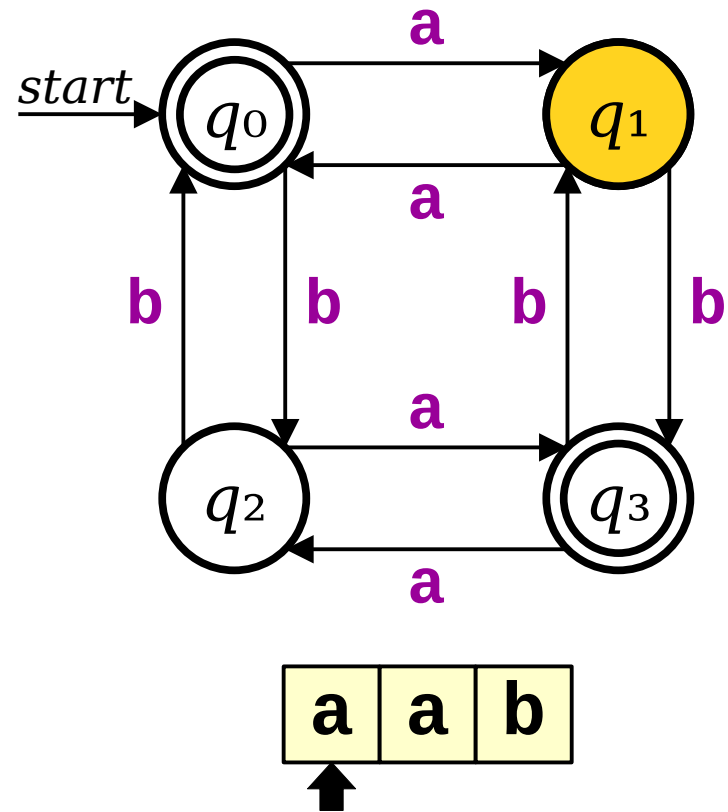
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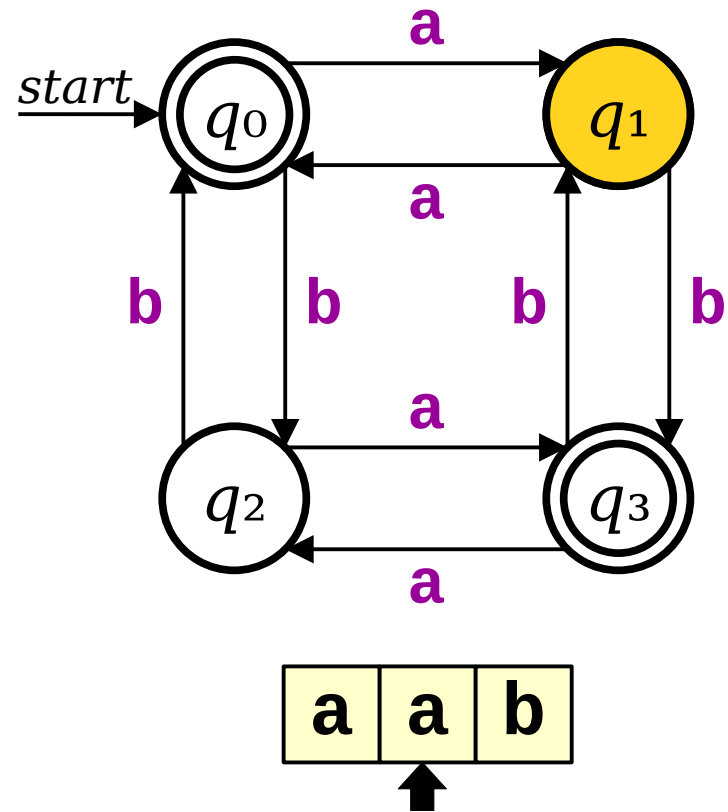
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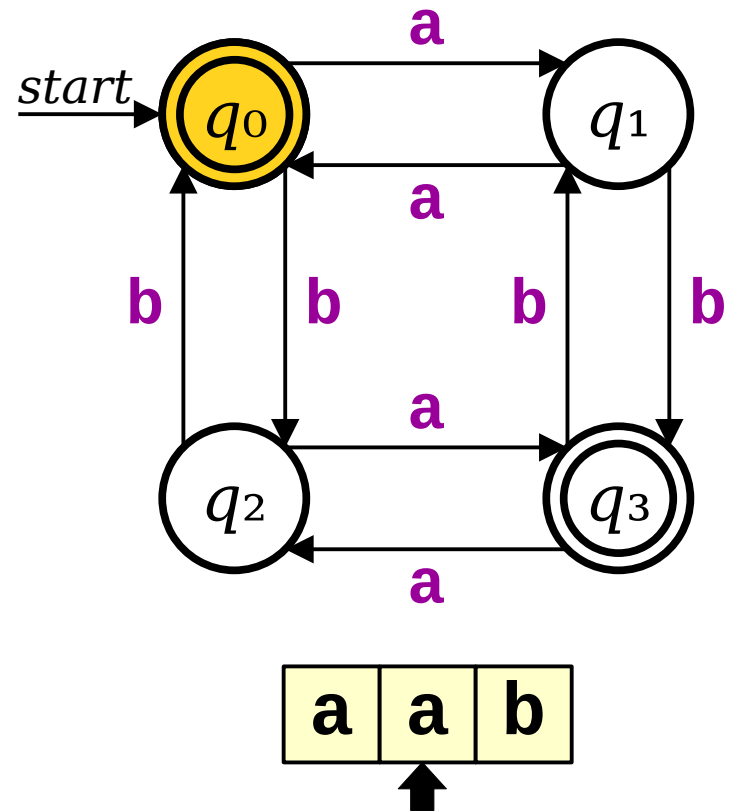
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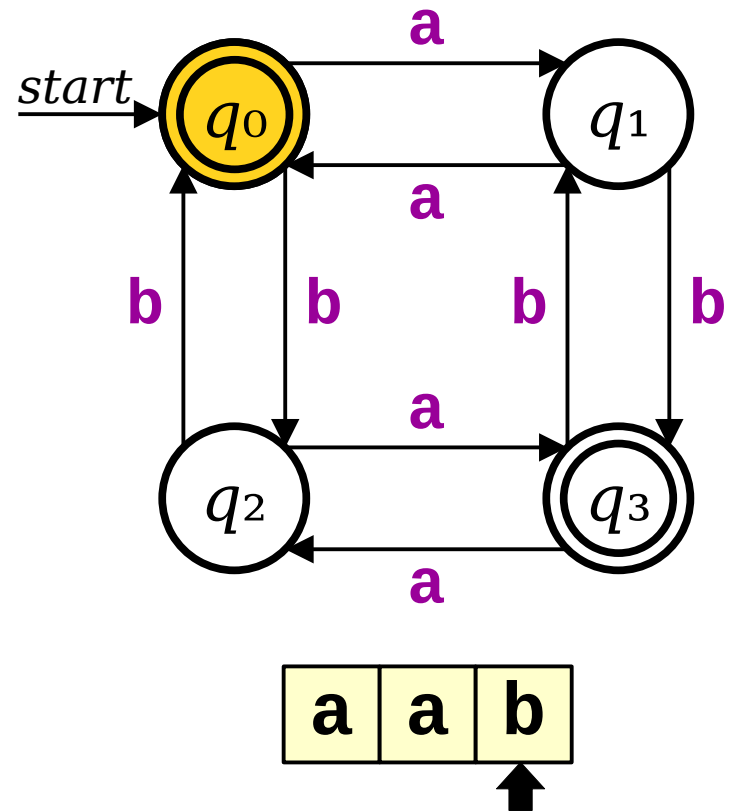
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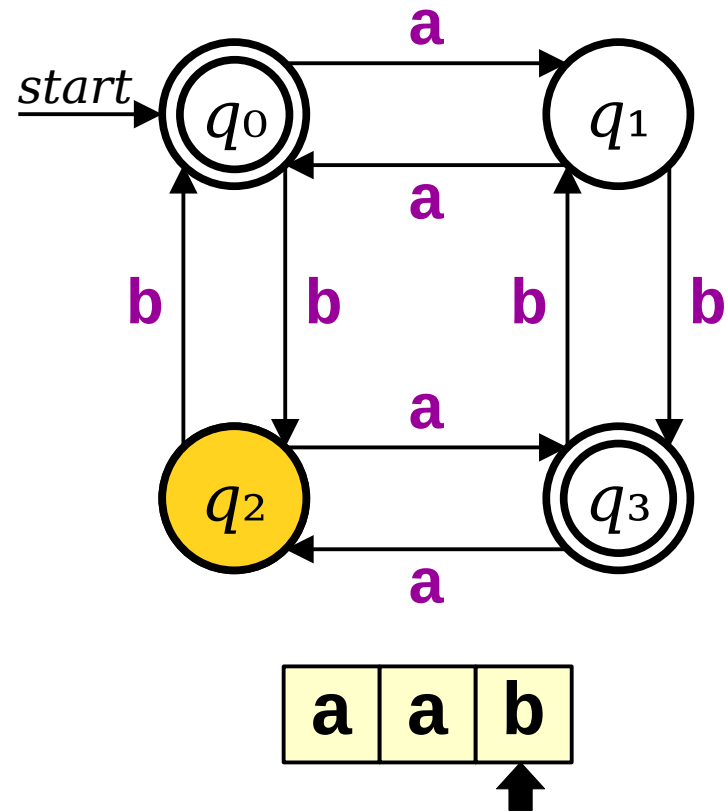
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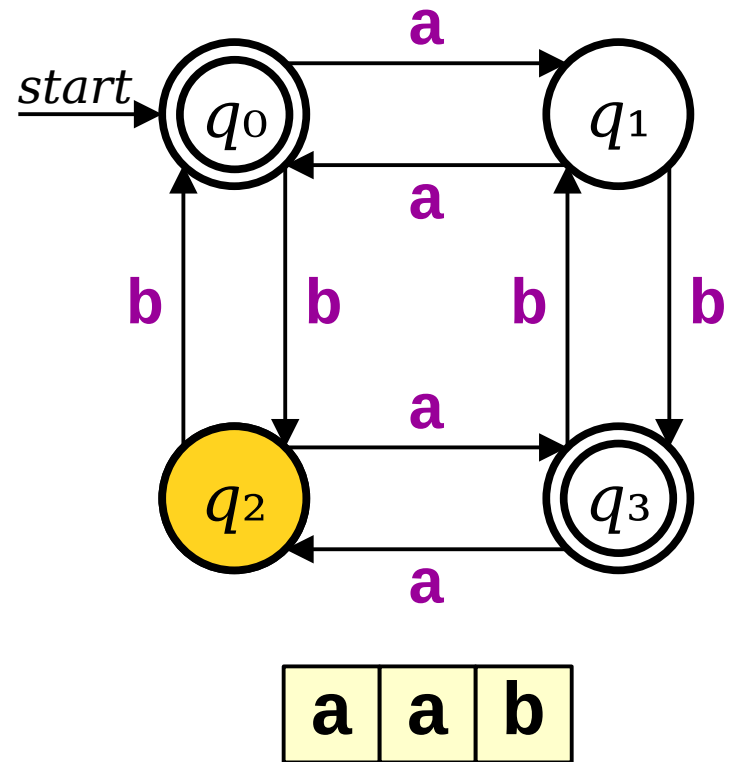
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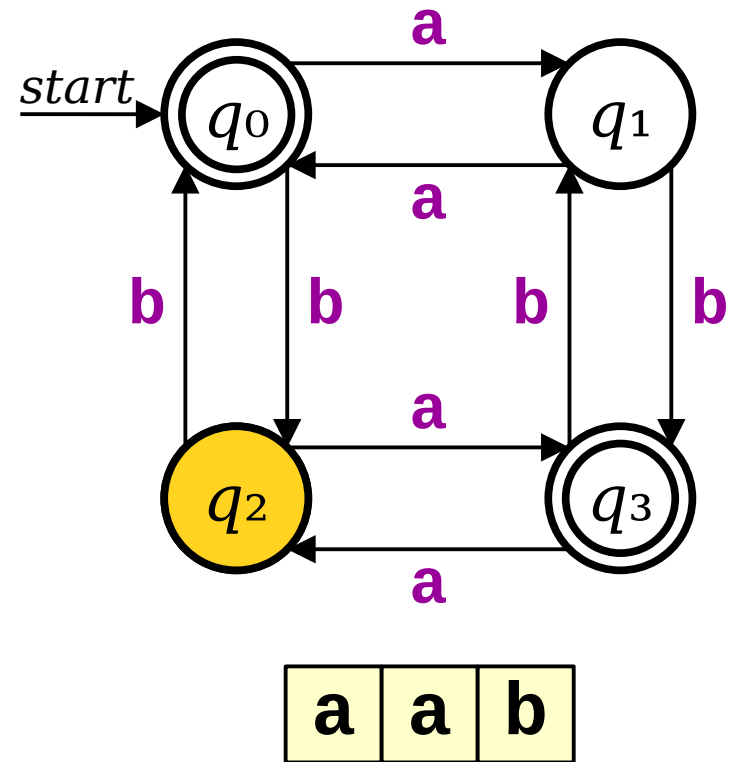
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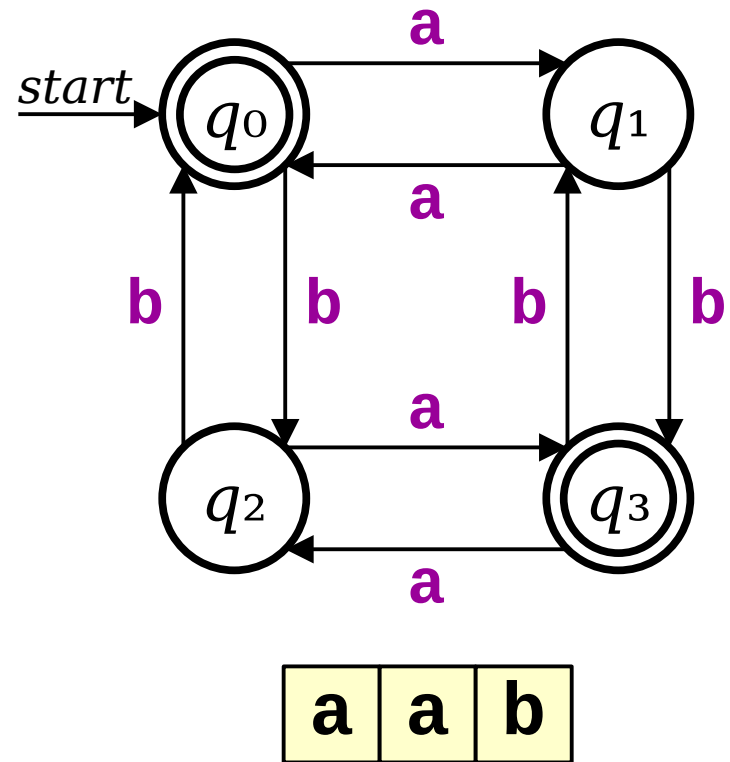
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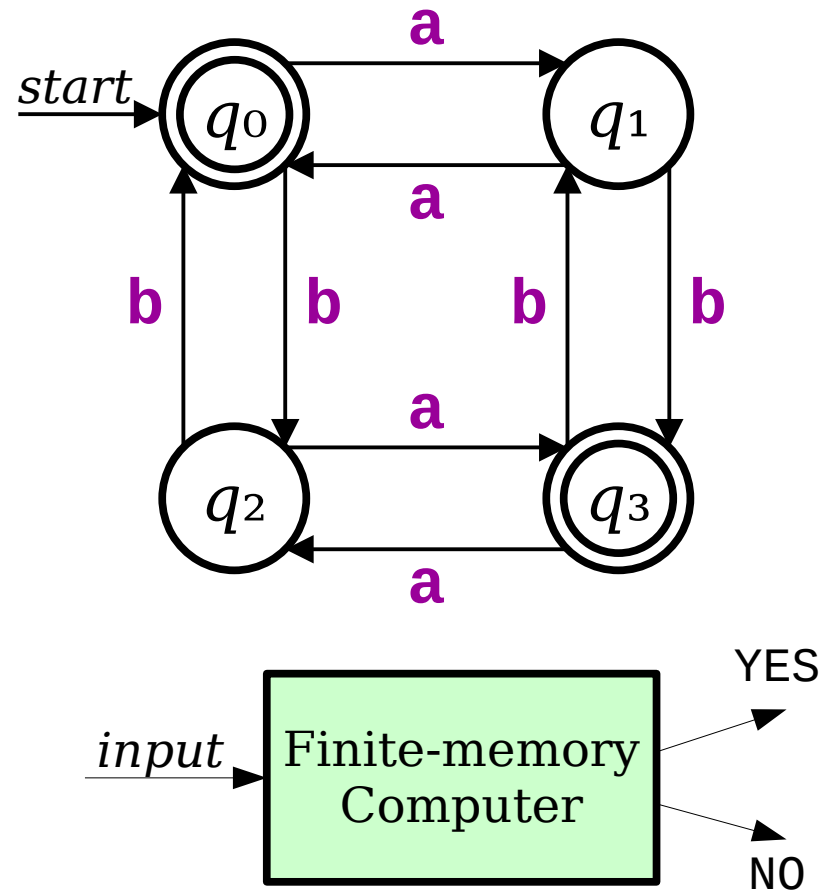
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# Finite Automata

- This type of computational device is called a **finite automaton** (plural: **finite automata**).
- Finite automata model computers where (1) memory is finite and (2) the computation produces as YES/NO answer.
- In other words, finite automata model predicates, and do so with a fixed, finite amount of memory.



# Formalizing Things

# Strings

- An **alphabet** is a finite, nonempty set of symbols called **characters**.
  - Typically, we use the symbol  $\Sigma$  to refer to an alphabet.
- A **string over an alphabet  $\Sigma$**  is a finite sequence of characters drawn from  $\Sigma$ .
- Example: Let  $\Sigma = \{a, b\}$ . Here are some strings over  $\Sigma$ :  
**a**      **aabaaabbabaaabbbb**      **abbababba**
- The **empty string** has no characters and is denoted  $\epsilon$ .
- Calling attention to an earlier point: since all strings are finite sequences of characters from  $\Sigma$ , you cannot have a string of infinite length.

# Languages

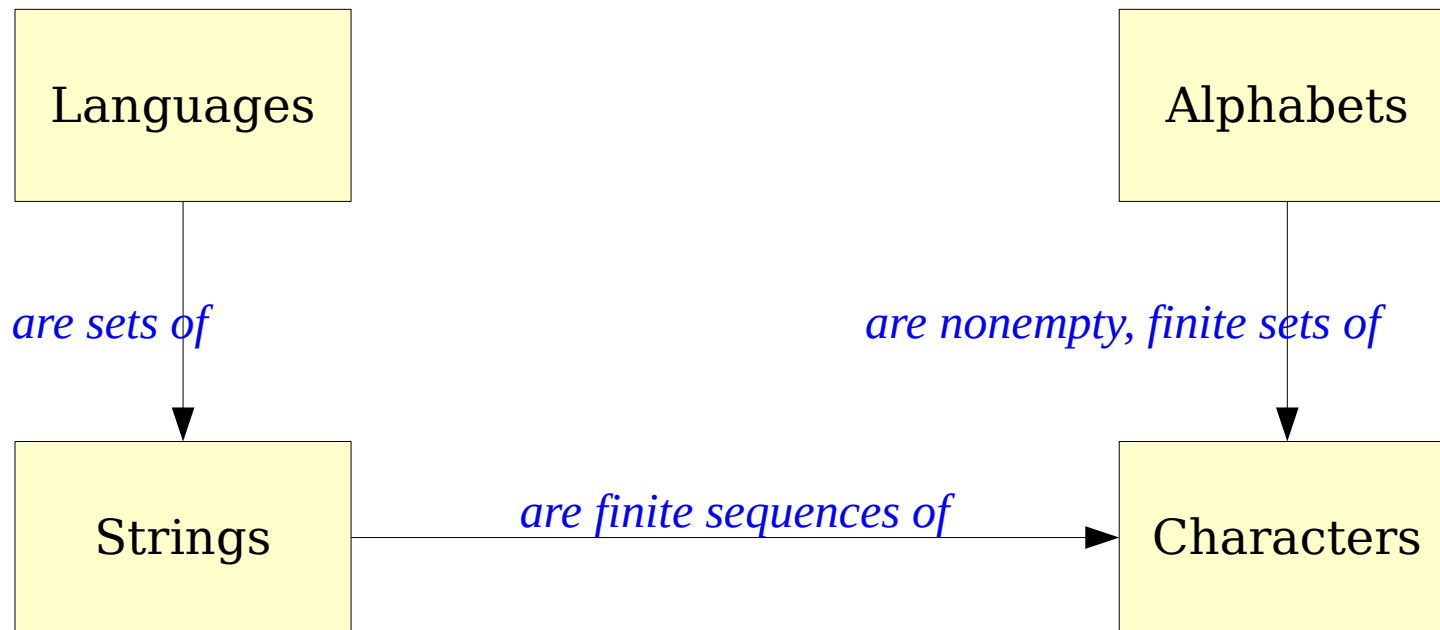
- A **formal language** is a set of strings.
- We say that  $L$  is a **language over  $\Sigma$**  if it is a set of strings over  $\Sigma$ .
- Example: The language of palindromes over  $\Sigma = \{a, b, c\}$  is the set
  - $\{\varepsilon, a, b, c, aa, bb, cc, aaa, aba, aca, bab, \dots\}$
- The set of all strings composed from letters in  $\Sigma$  is denoted  $\Sigma^*$ .
- Formally, we say that  $L$  is a language over  $\Sigma$  if  $L \subseteq \Sigma^*$ .

# Mathematical Lookalikes

- We now have  $\in$ ,  $\varepsilon$ ,  $\Sigma$ , and  $\Sigma^*$ . Yikes!
- The symbol  $\in$  is the ***element-of*** relation.
- The symbol  $\varepsilon$  is the ***empty string***.
- The symbol  $\Sigma$  denotes an ***alphabet***.
- The expression  $\Sigma^*$  means “all strings that can be made from characters in  $\Sigma$ .”
- That lets us write things like  
$$\text{We have } \varepsilon \in \Sigma^*, \text{ but } \varepsilon \notin \Sigma.$$
- Ever get confused? ***Just ask!***

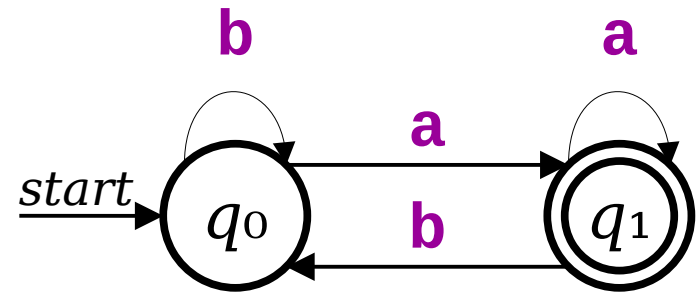
# The Cast of Characters

- **Languages** are sets of strings.
- **Strings** are finite sequences of characters.
- **Characters** are individual symbols.
- **Alphabets** are sets of characters.



# Finite Automata and Languages

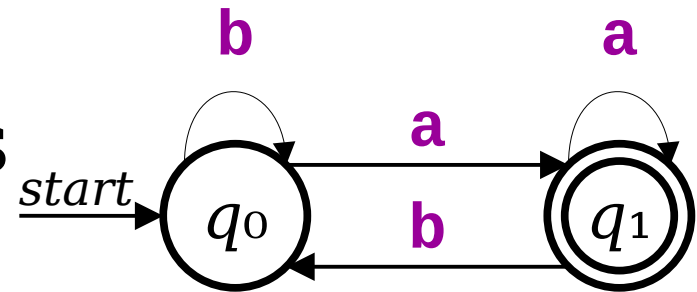
- Let  $A$  be an automaton that processes strings drawn from an alphabet  $\Sigma$ .
- The **language of  $A$** , denoted  $\mathcal{L}(A)$ , is the set of strings over  $\Sigma$  that  $A$  accepts:



$$\mathcal{L}(A) = \{ w \in \Sigma^* \mid A \text{ accepts } w \}$$

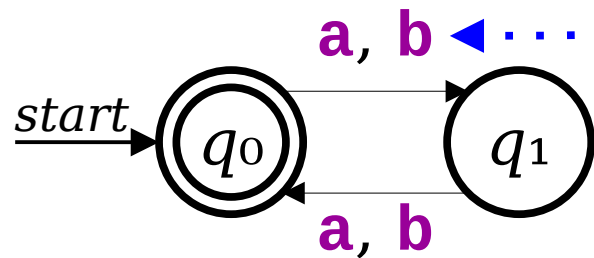
# Finite Automata and Languages

- Let  $D$  be the automaton shown to the right. It processes strings over  $\{\mathbf{a}, \mathbf{b}\}$ .
- Notice that  $D$  accepts all strings of  $\mathbf{a}$ 's and  $\mathbf{b}$ 's that end in  $\mathbf{a}$  and rejects everything else.
- So  $\mathcal{L}(D) = \{ w \in \{\mathbf{a}, \mathbf{b}\}^* \mid w \text{ ends in } \mathbf{a} \}$ .

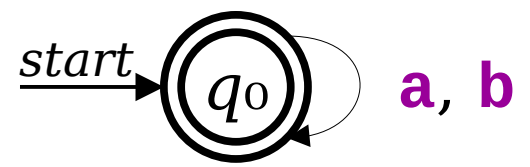
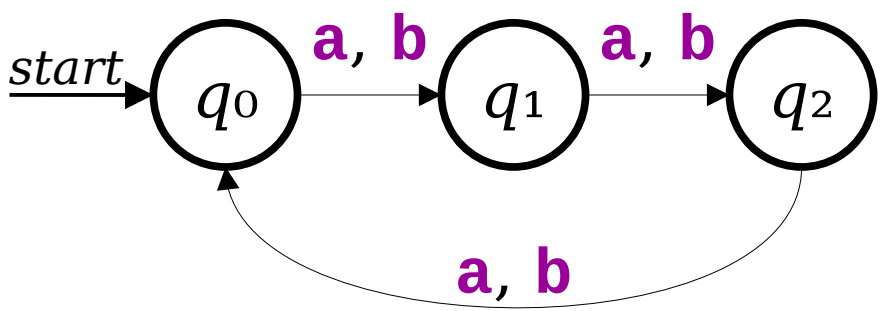
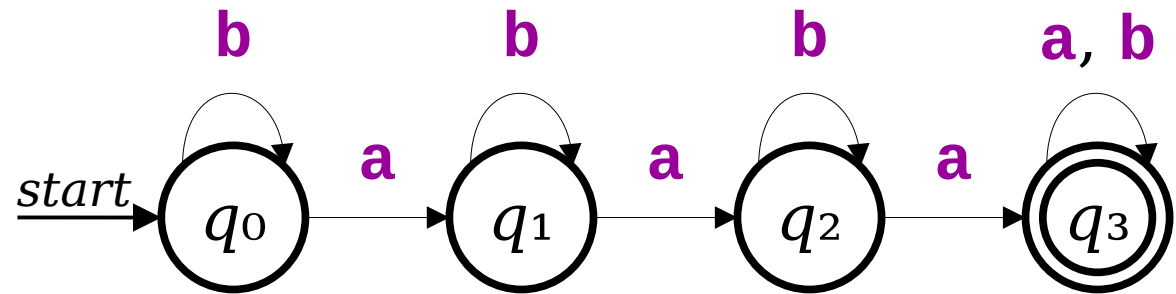


$$\mathcal{L}(A) = \{ w \in \Sigma^* \mid A \text{ accepts } w \}$$

# Finite Automata and Languages



This means “take this transition if you see an **a** or a **b**.”



$$\mathcal{L}(A) = \{ w \in \Sigma^* \mid A \text{ accepts } w \}$$

# The Story So Far

- A ***finite automaton*** is a collection of ***states*** joined by ***transitions***.
- Some state is designated as the ***start state***.
- Some number of states are designated as ***accepting states***.
- The automaton processes a string by beginning in the start state and following the indicated transitions.
- If the automaton ends in an accepting state, it ***accepts*** the input.
- Otherwise, the automaton ***rejects*** the input.
- The ***language*** of an automaton is the set of strings it accepts.

Let's take a quick break!

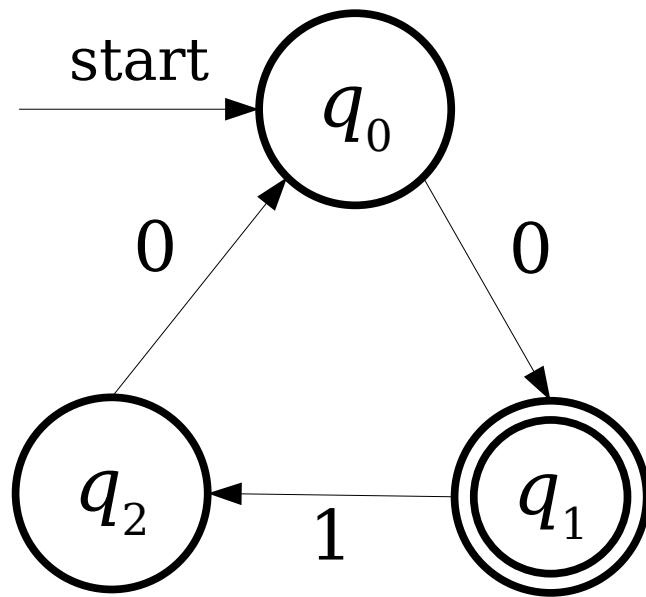
**Time-Out For Announcements!**

# PS3 Solutions Released

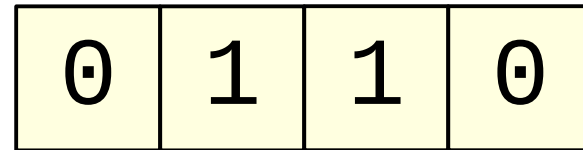
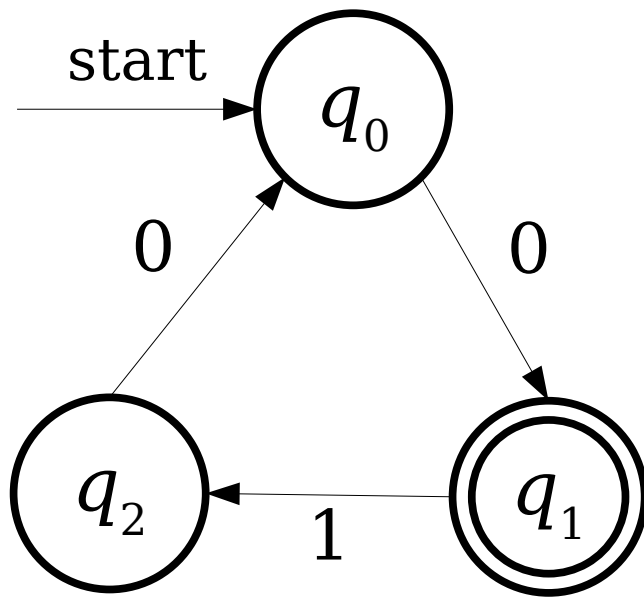
- Solutions to Problem Set Three are now available on the course website.
- We generally don't release solutions to autograded problems. If you have any questions about those, please ping us privately over EdStem or come talk to us in our office hours.
- We're aiming to have things graded by Wednesday before class so that you can review your feedback before the midterm exam.

Back to CS103!

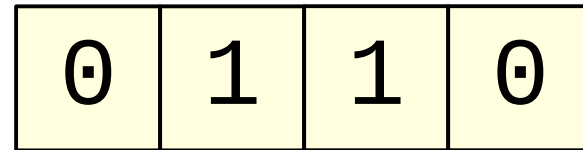
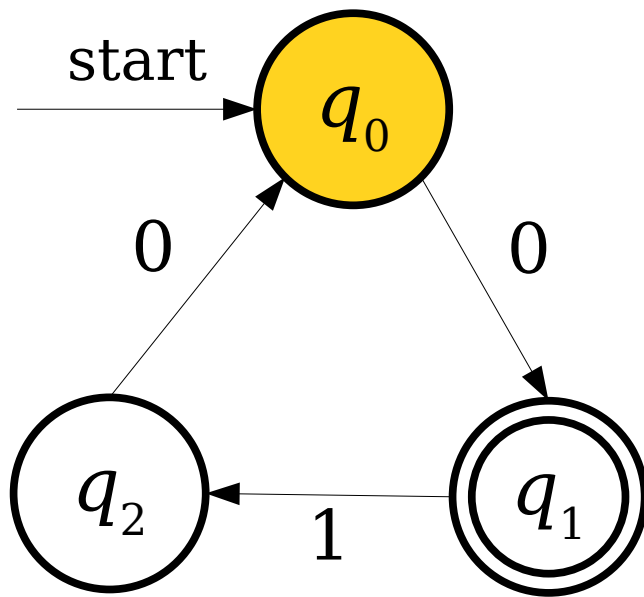
# A Small Problem



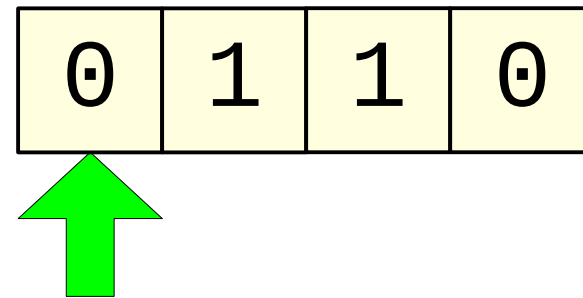
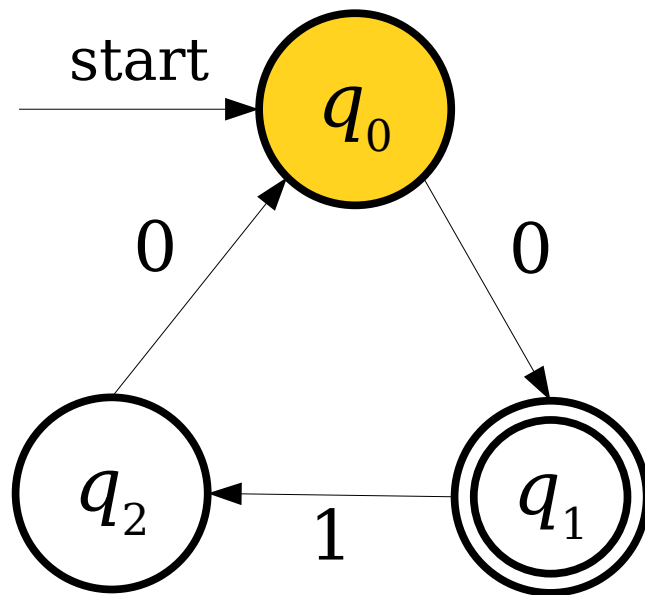
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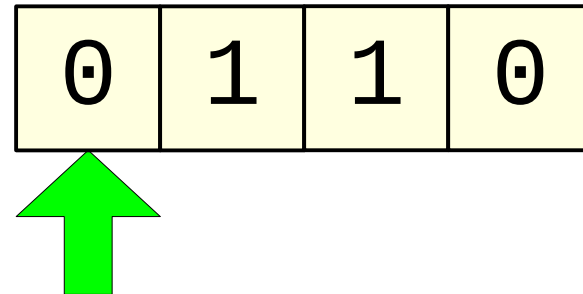
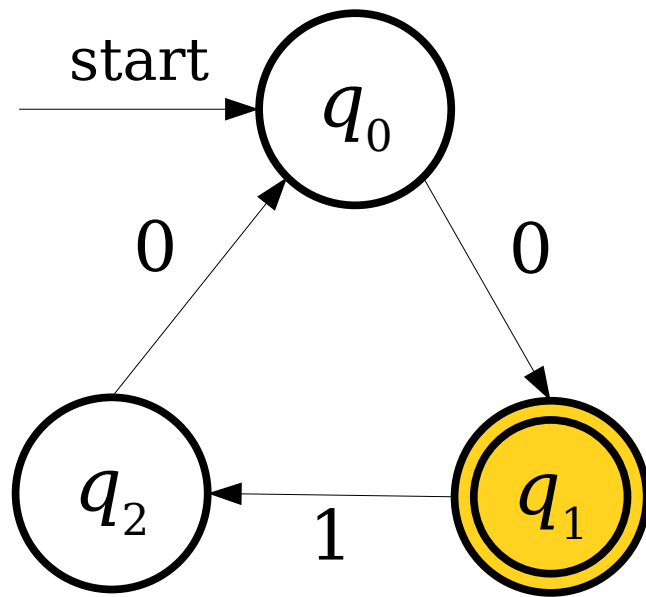
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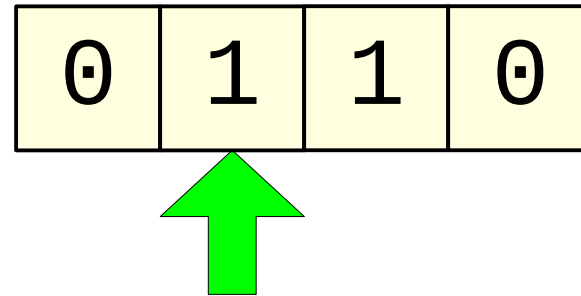
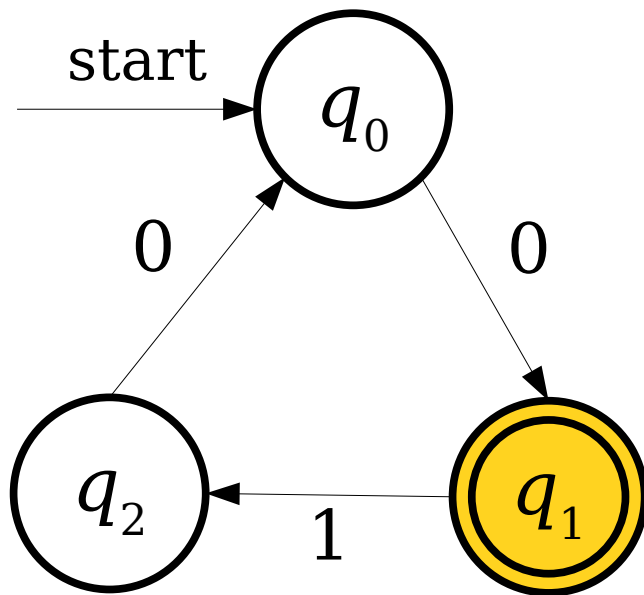
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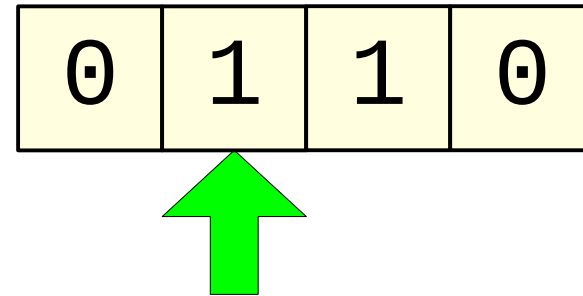
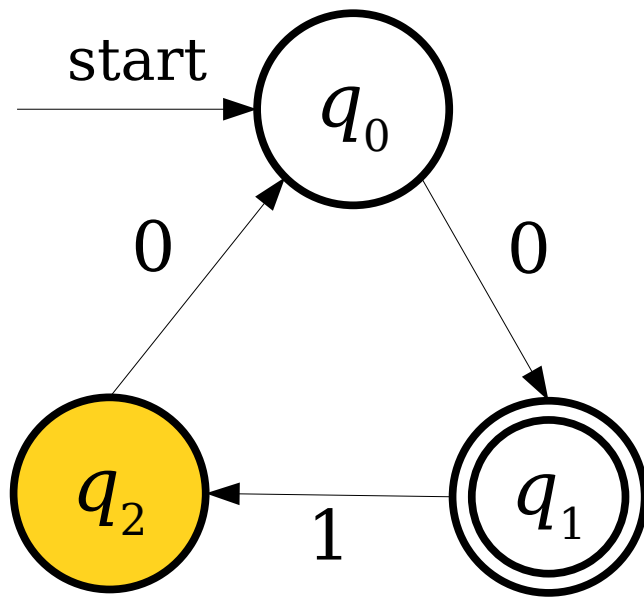
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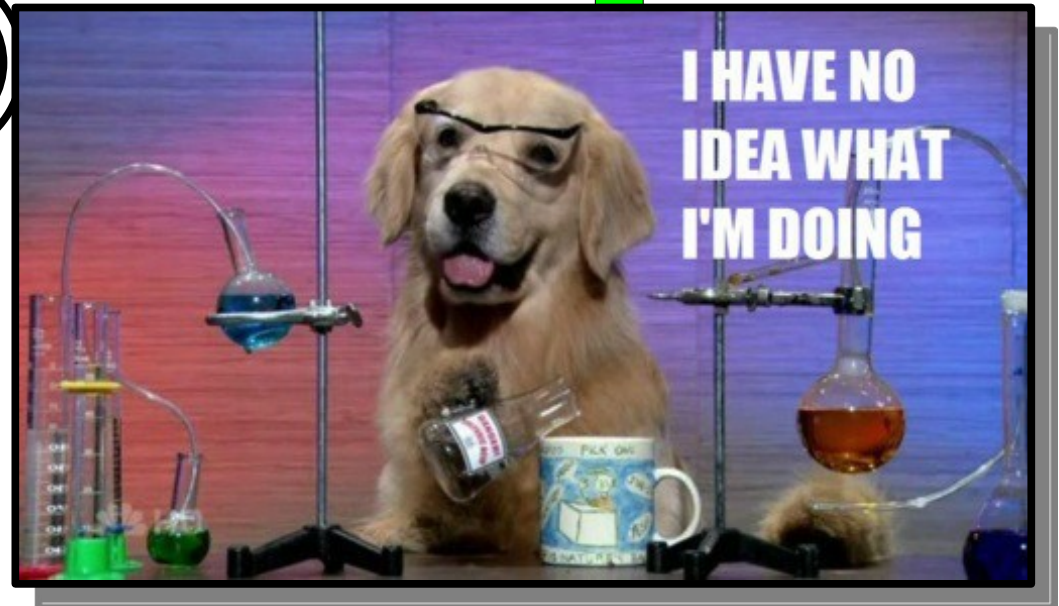
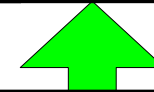
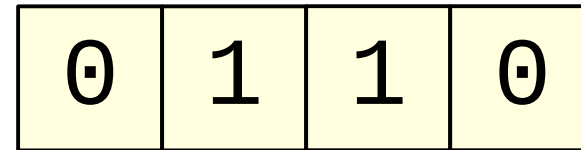
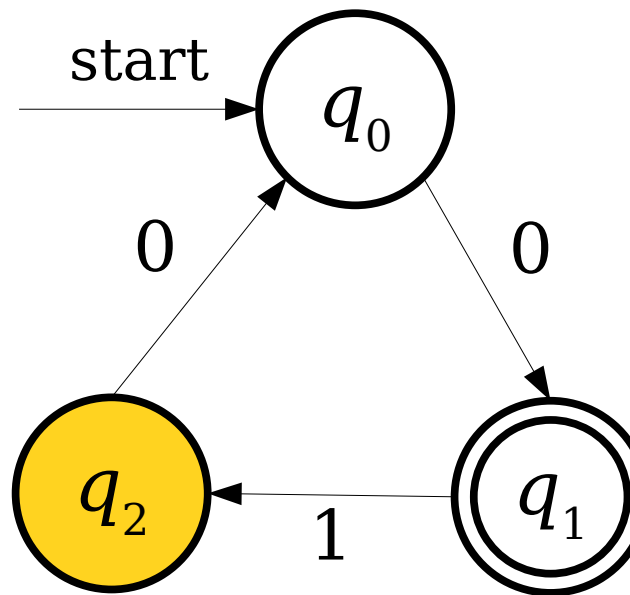
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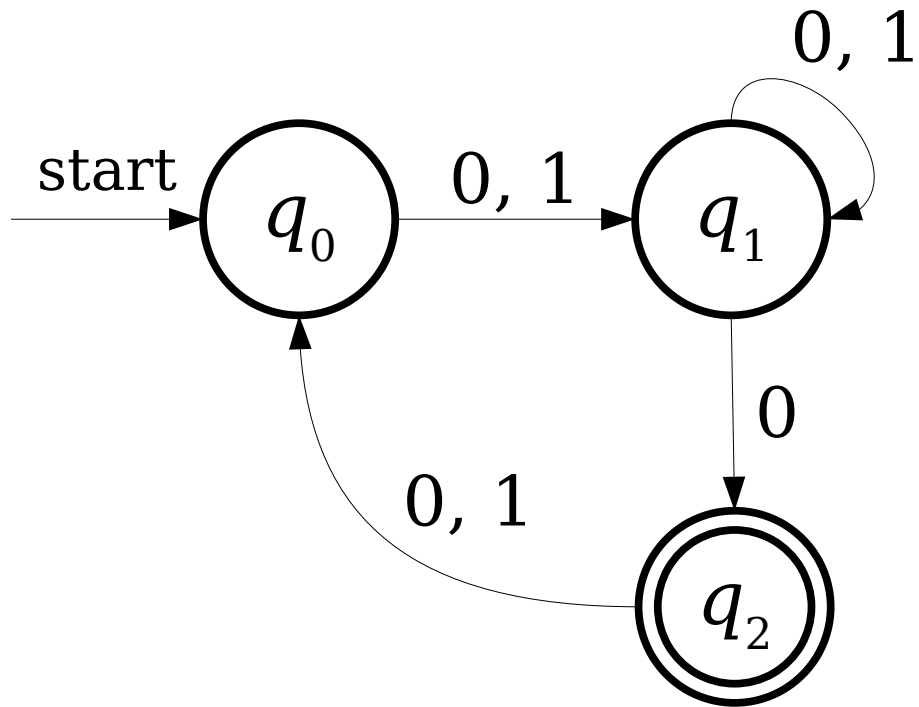
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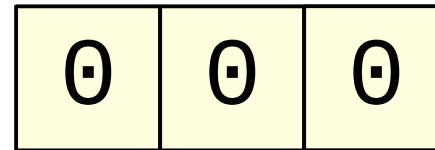
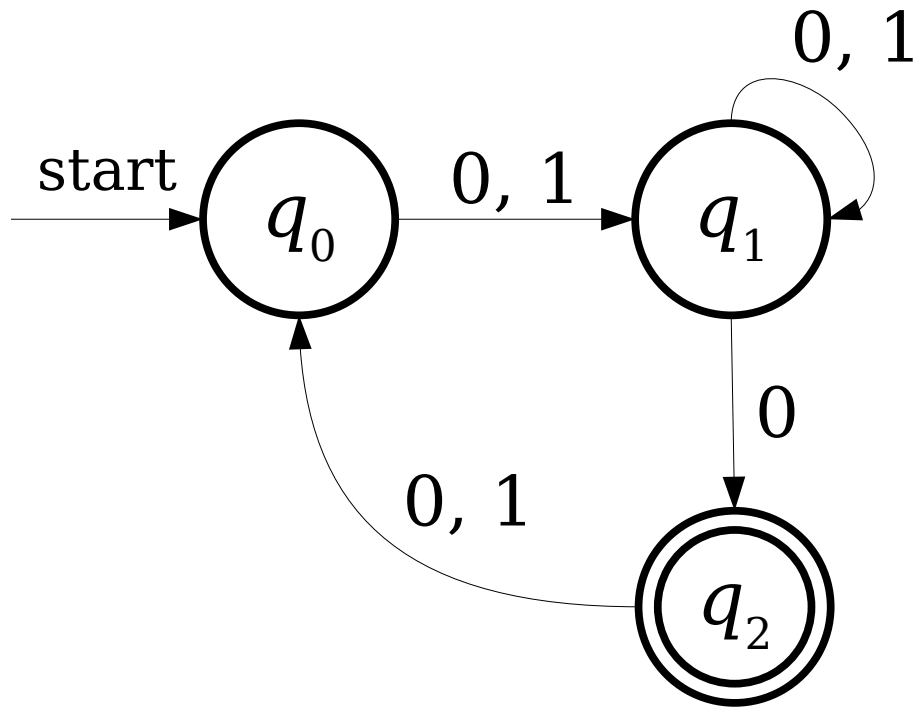
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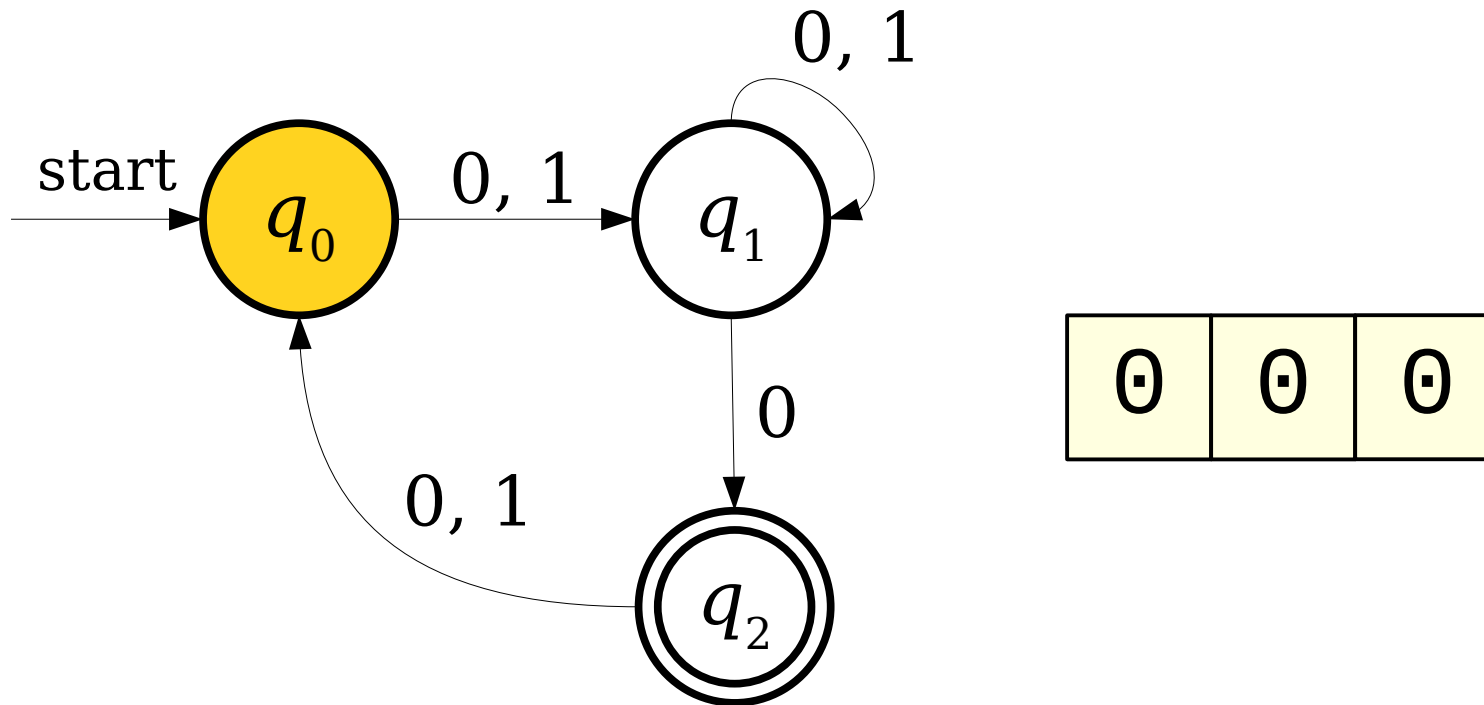
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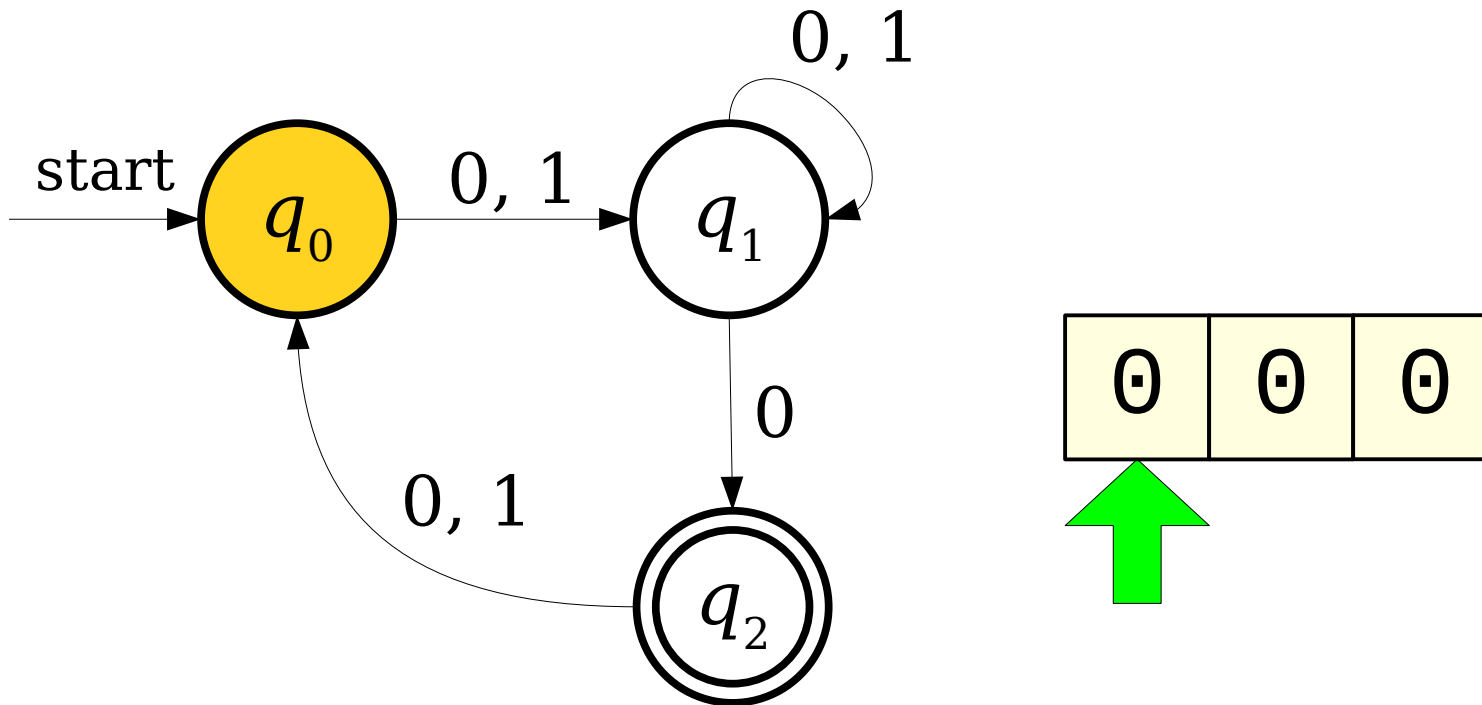
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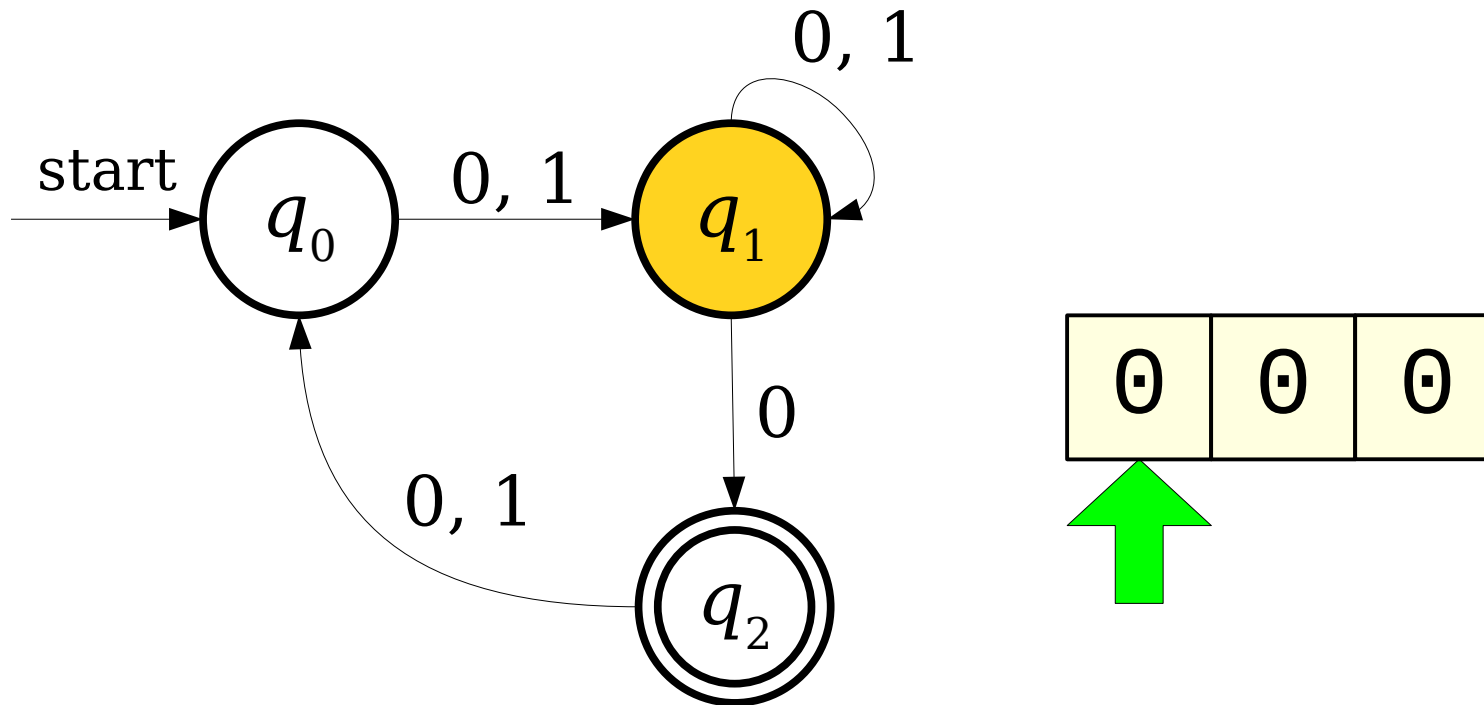
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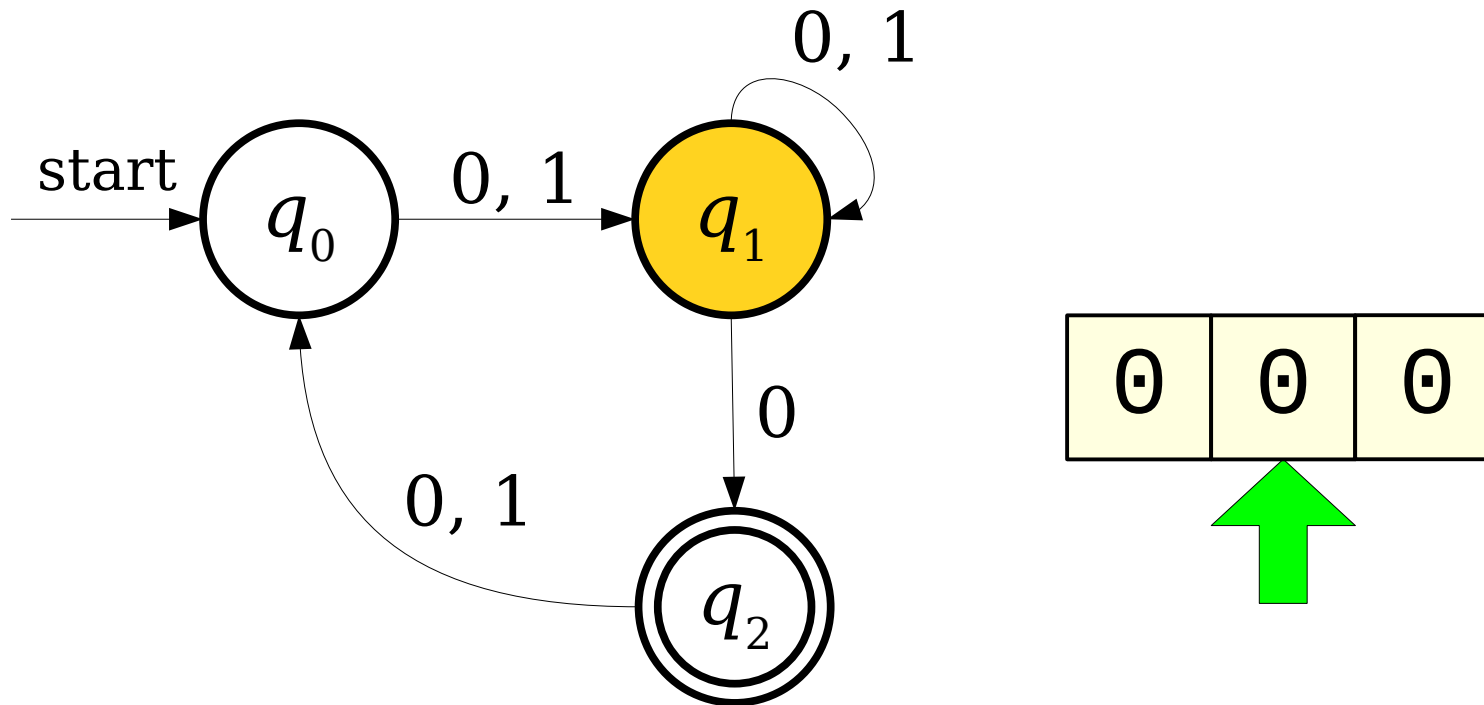
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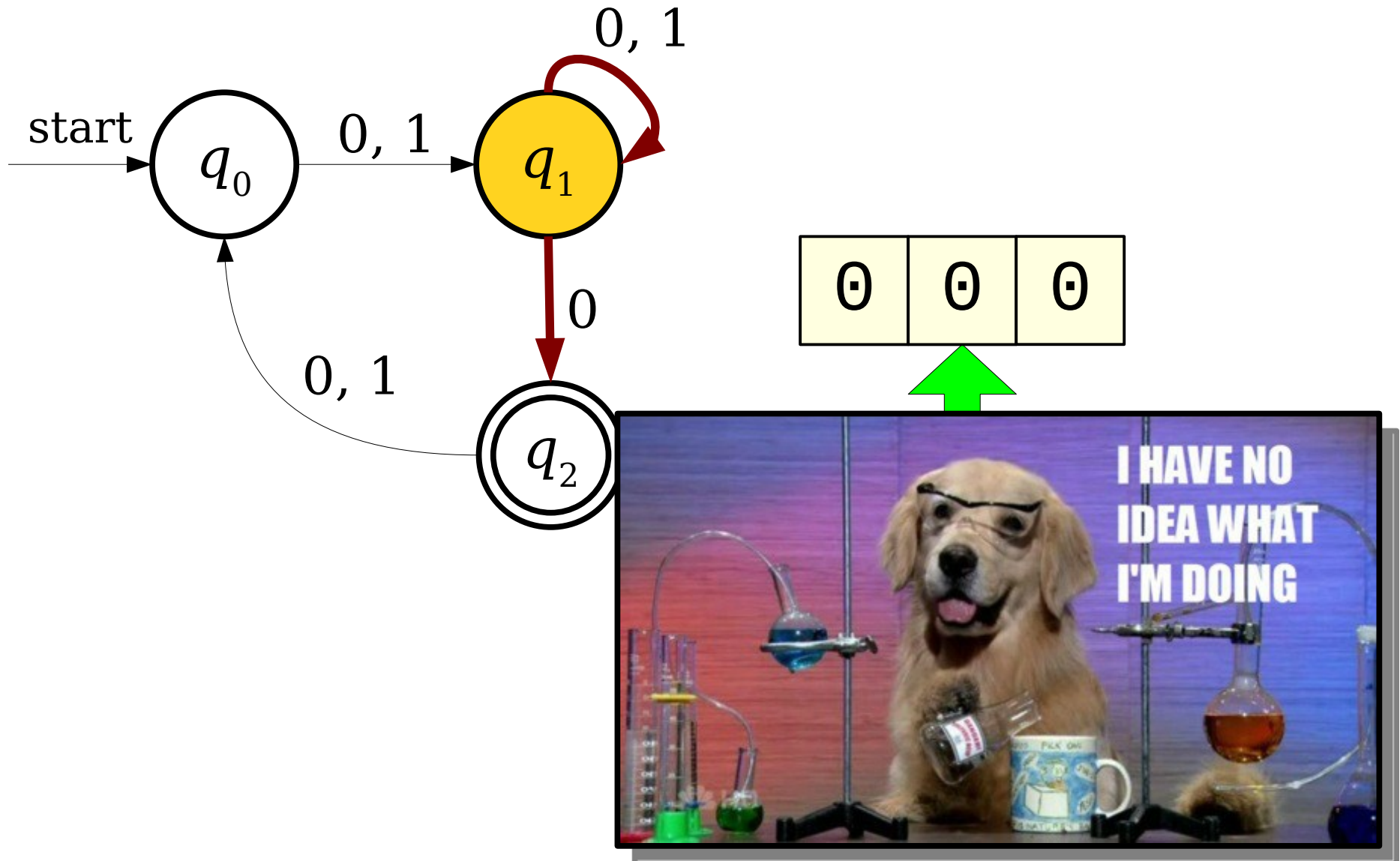
# Another Small Problem



# Another Small Problem



# Another Small Problem



# The Need for Formalism

- In order to reason about the limits of what finite automata can and cannot do, we need to formally specify their behavior in *all* cases.
- All of the following need to be defined or disallowed:
  - What happens if there is no transition out of a state on some input?
  - What happens if there are *multiple* transitions out of a state on some input?

# DFAs

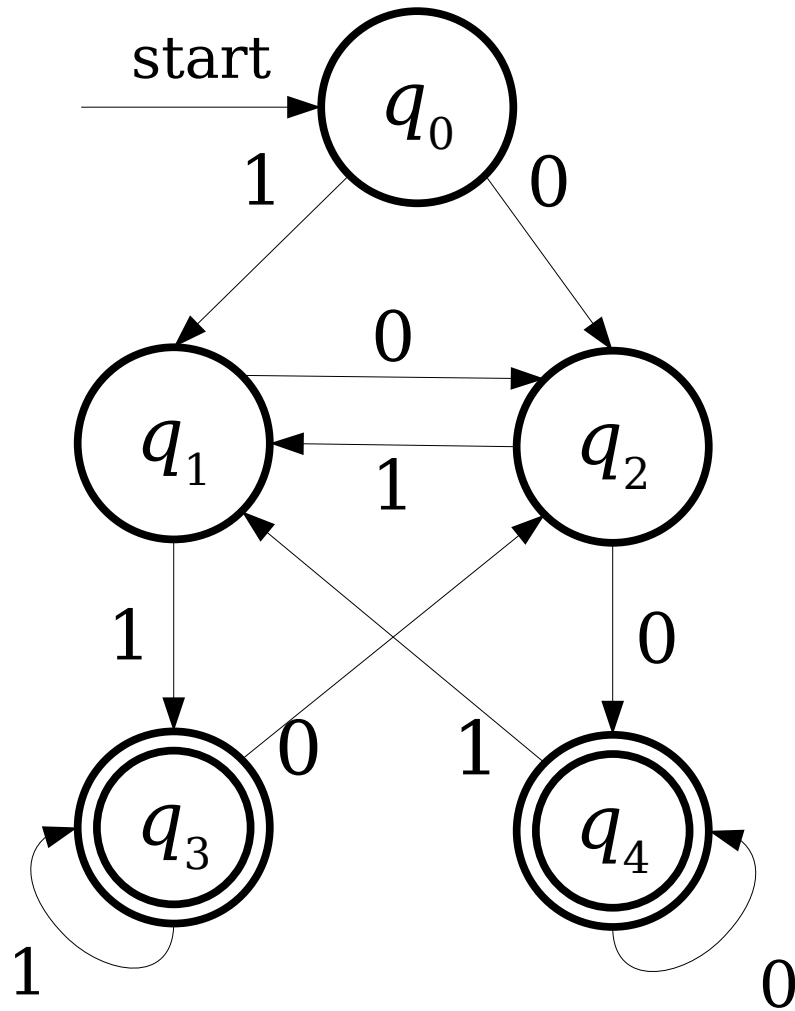
- A ***DFA*** is a
  - ***D***eterministic
  - ***F***inite
  - ***A***utomaton
- DFAs are the simplest type of automaton that we will see in this course.

# DFA's

- A DFA is defined relative to some alphabet  $\Sigma$ .
- For each state in the DFA, there must be *exactly one* transition defined for each symbol in  $\Sigma$ .
  - This is the “deterministic” part of DFA.
- There is a unique start state.
- There are zero or more accepting states.

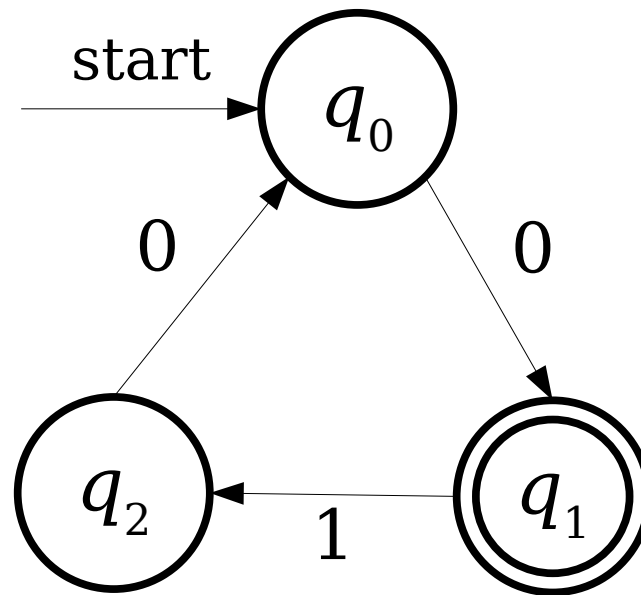
Is this a DFA over  $\{0, 1\}$ ?

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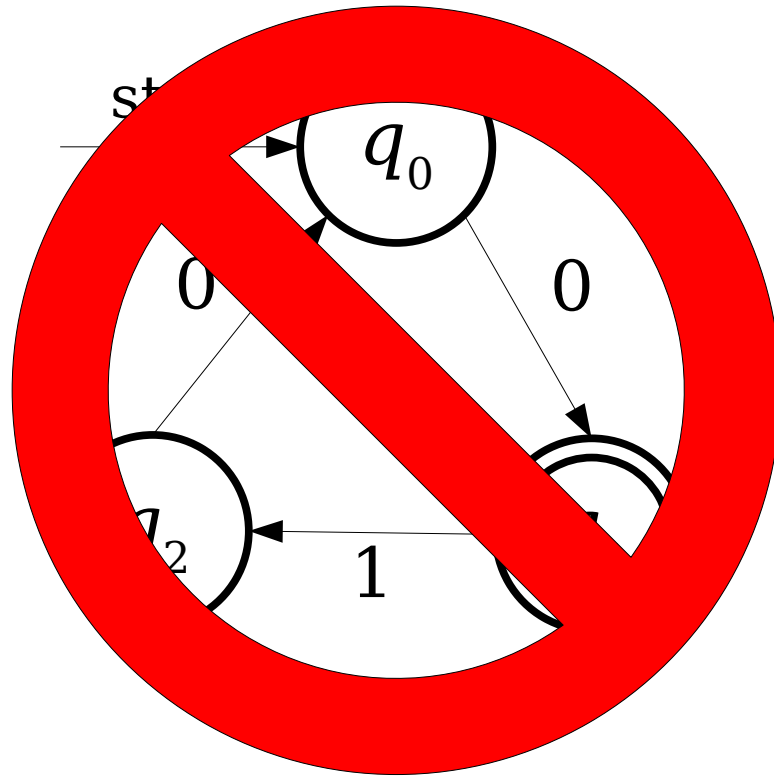


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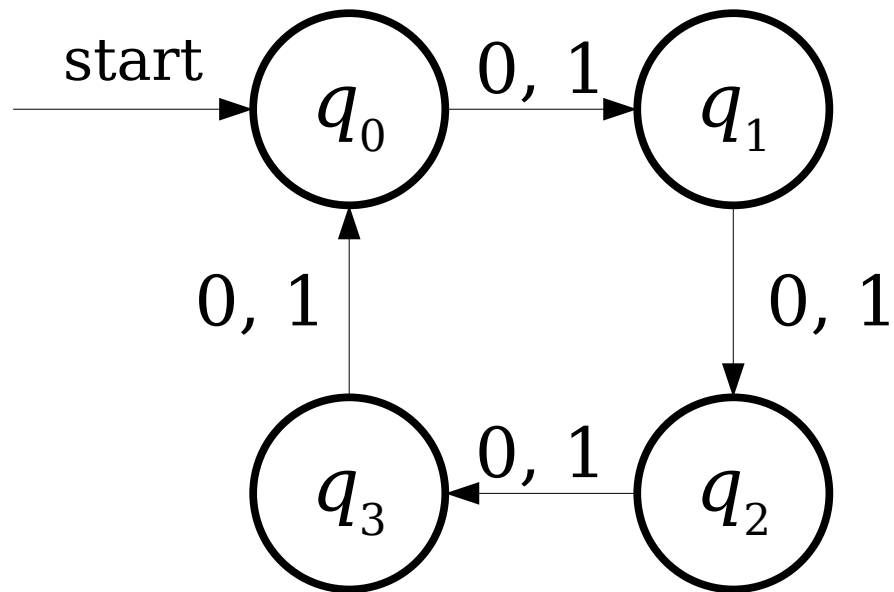


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Is this a DFA over  $\{0, 1\}$ ?

Is this a DFA?



Is this a DFA?



**D**rinking **F**amily of **A**ardvarks

# Designing DFAs

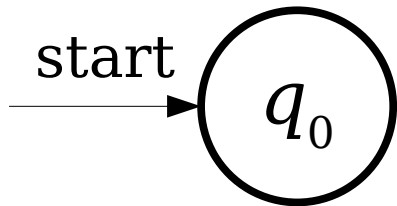
- At each point in its execution, the DFA can only remember what state it is in.
- ***DFA Design Tip:*** Build each state to correspond to some piece of information you need to remember.
  - Each state acts as a “memento” of what you're supposed to do next.
  - Only finitely many different states means only finitely many different things the machine can remember.

# Recognizing Languages with DFAs

$L = \{ w \in \{a, b\}^* \mid \text{the number of } b\text{'s in } w \text{ is congruent to two modulo three} \}$

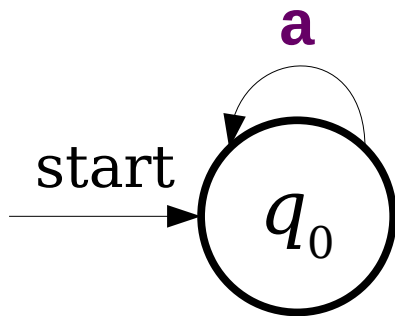
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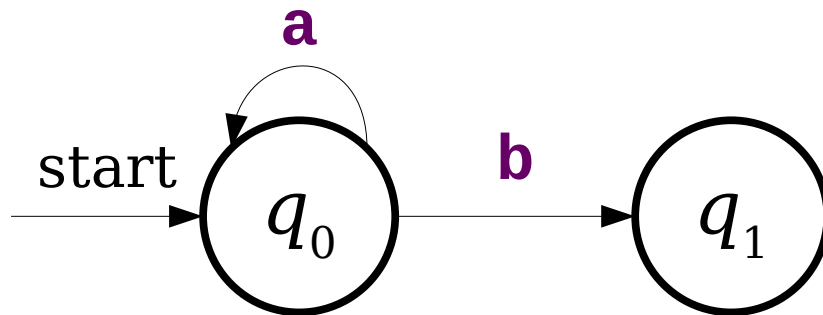
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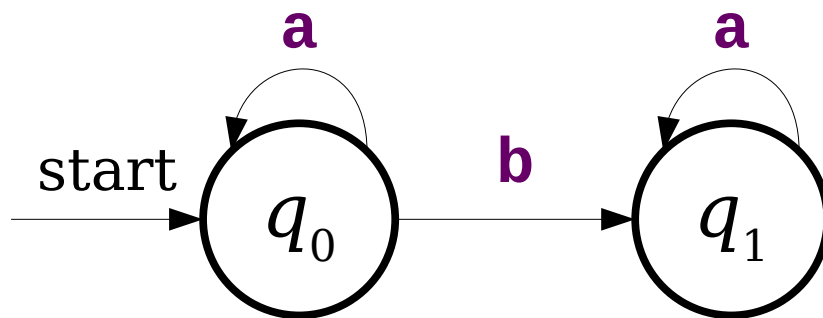
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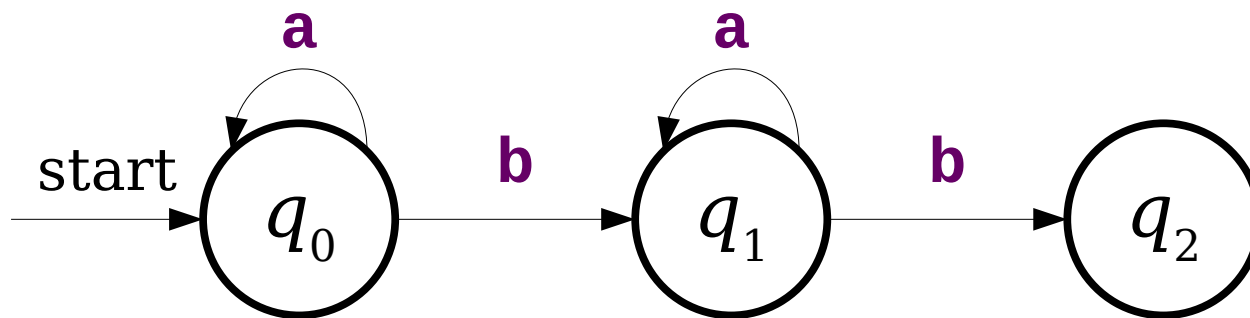
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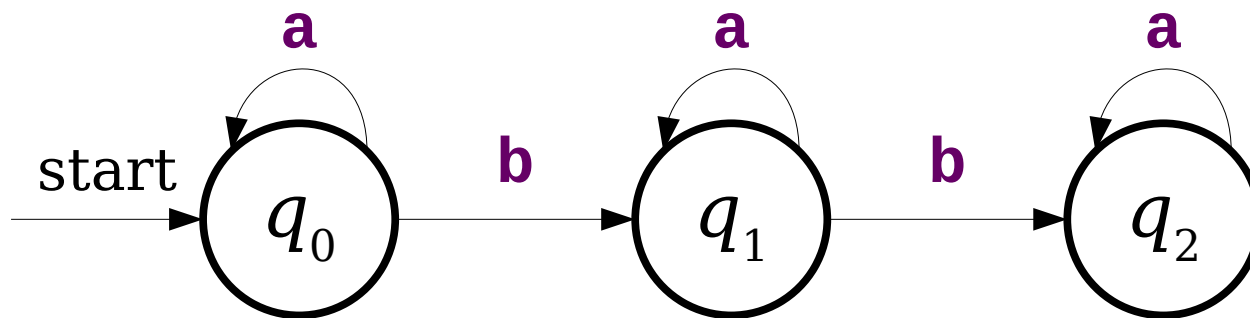
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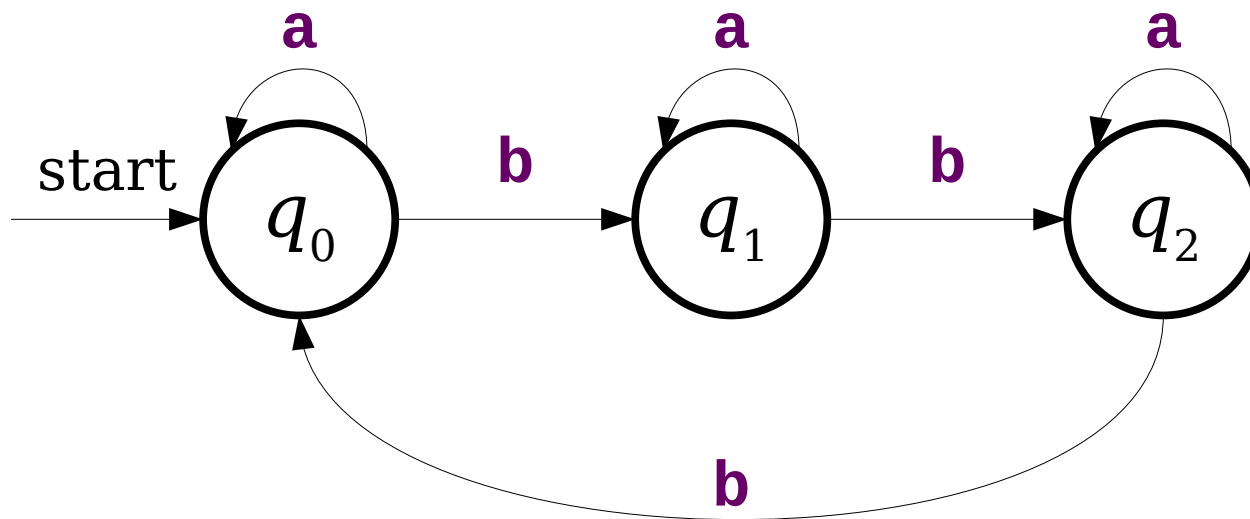
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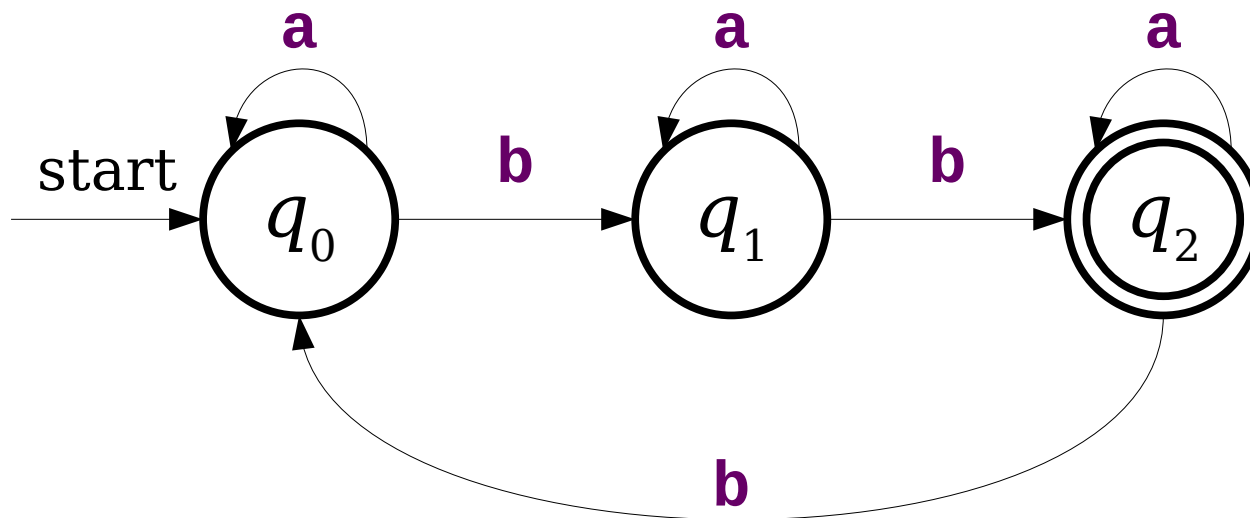
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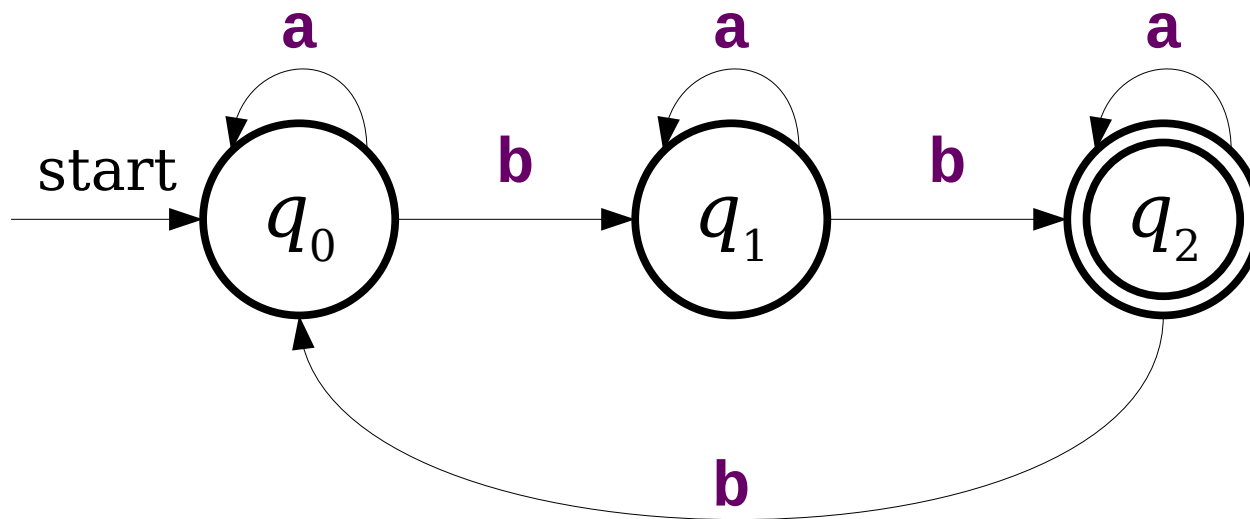
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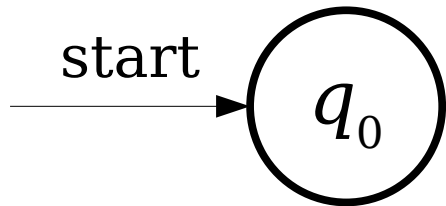
Each state remembers the remainder of the number of **b**s seen so far modulo three.

# Recognizing Languages with DFAs

$L = \{ w \in \{a, b\}^* \mid w \text{ contains } aa \text{ as a substring} \}$

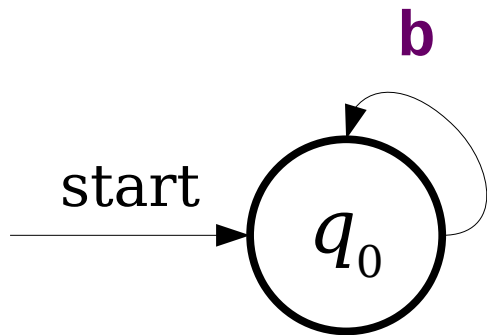
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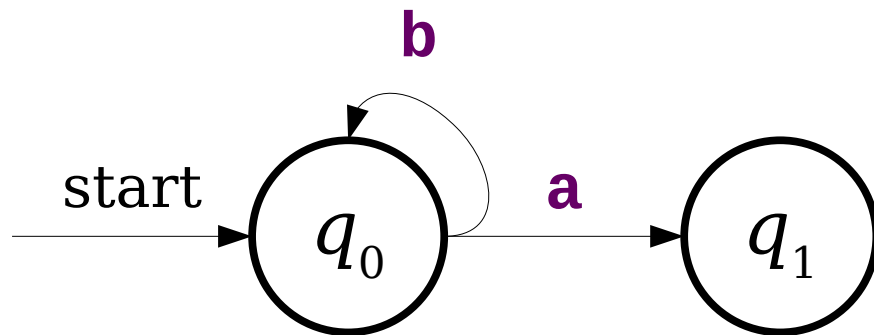
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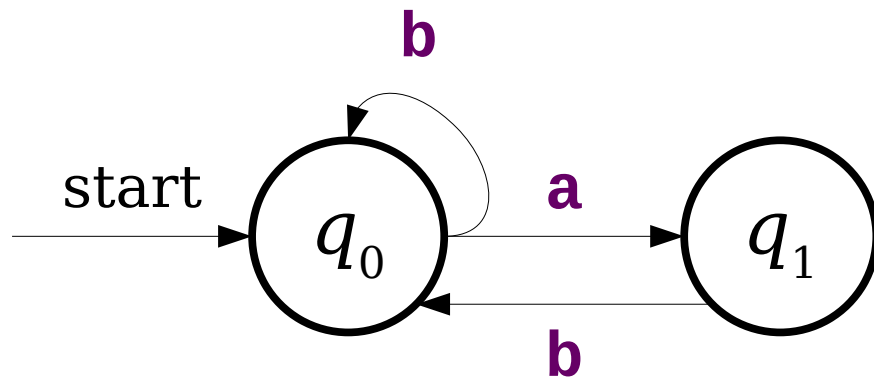
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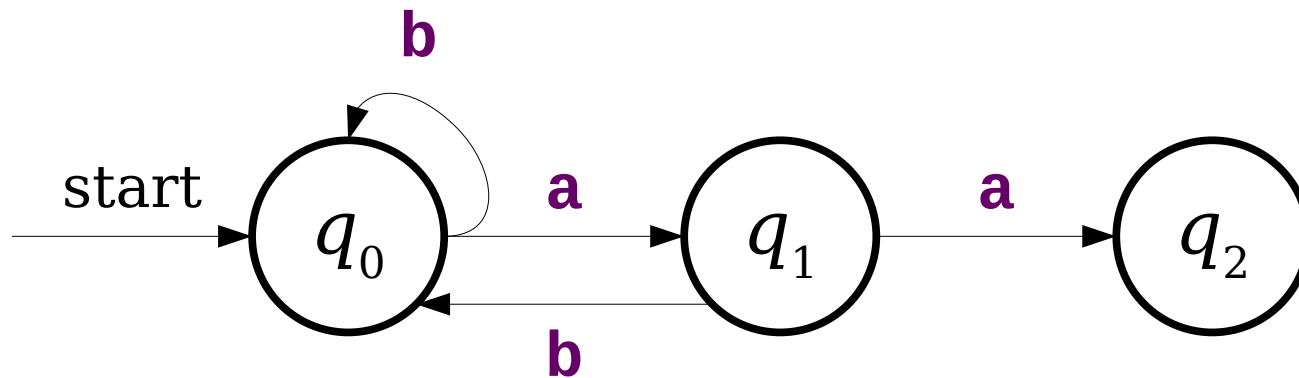
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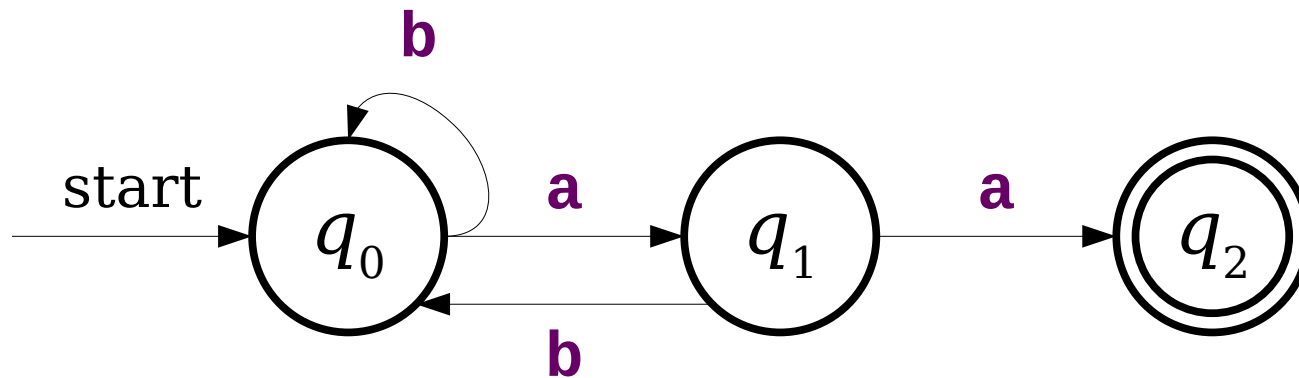
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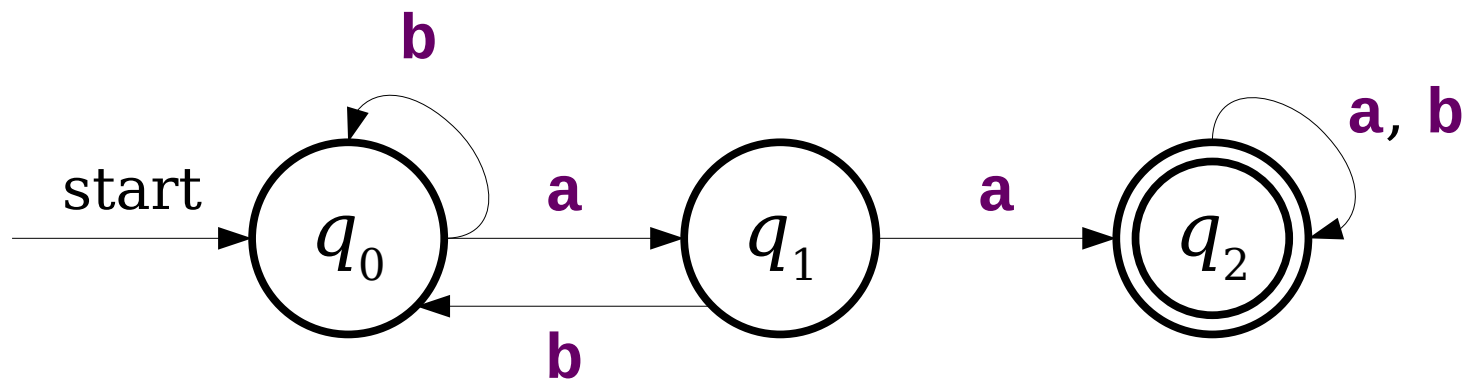
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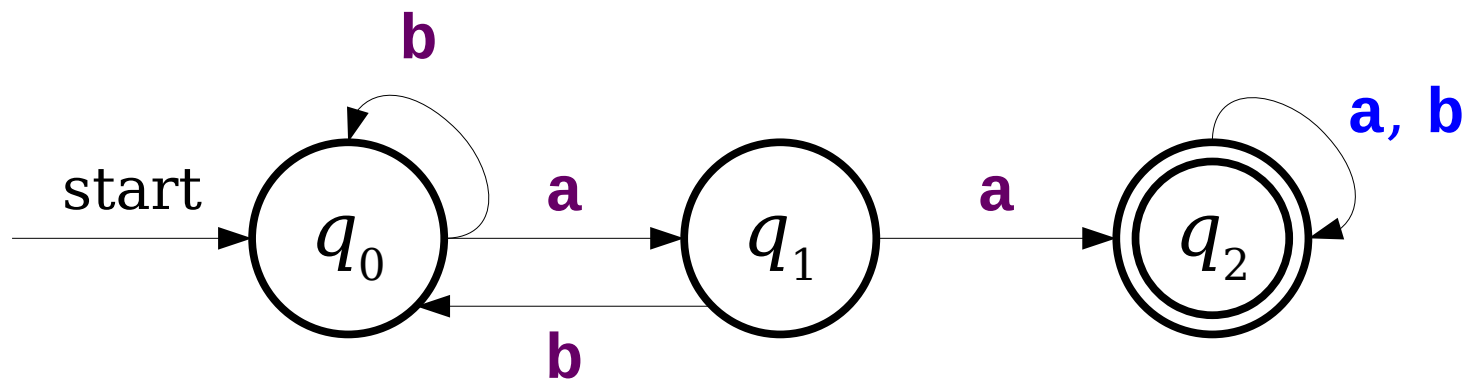
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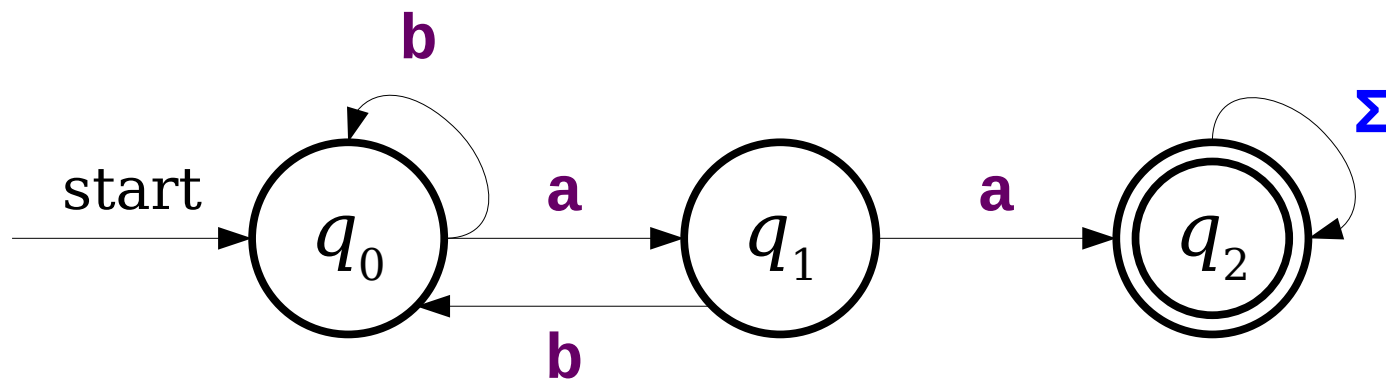
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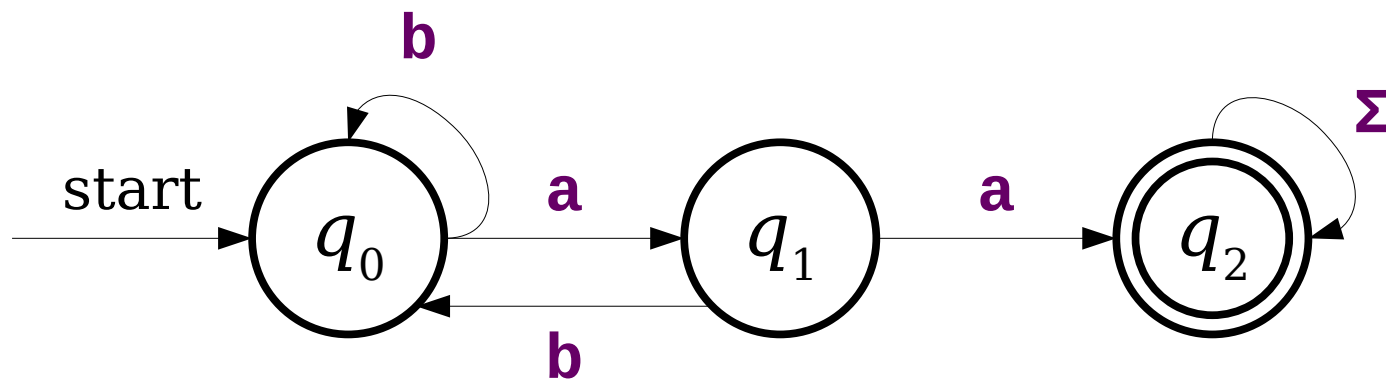
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# Recognizing Languages with DFAs

$L = \{ w \in \{a, b\}^* \mid w \text{ contains } aa \text{ as a substring} \}$



# More Elaborate DFAs

$L = \{ w \in \{a, *, /\}^* \mid w \text{ represents a C-style comment} \}$

Let's have the **a** symbol be a placeholder for "some character that isn't a star or slash."

Let's design a DFA for C-style comments. Those are the ones that start with `/*` and end with `*/`.

Accepted:

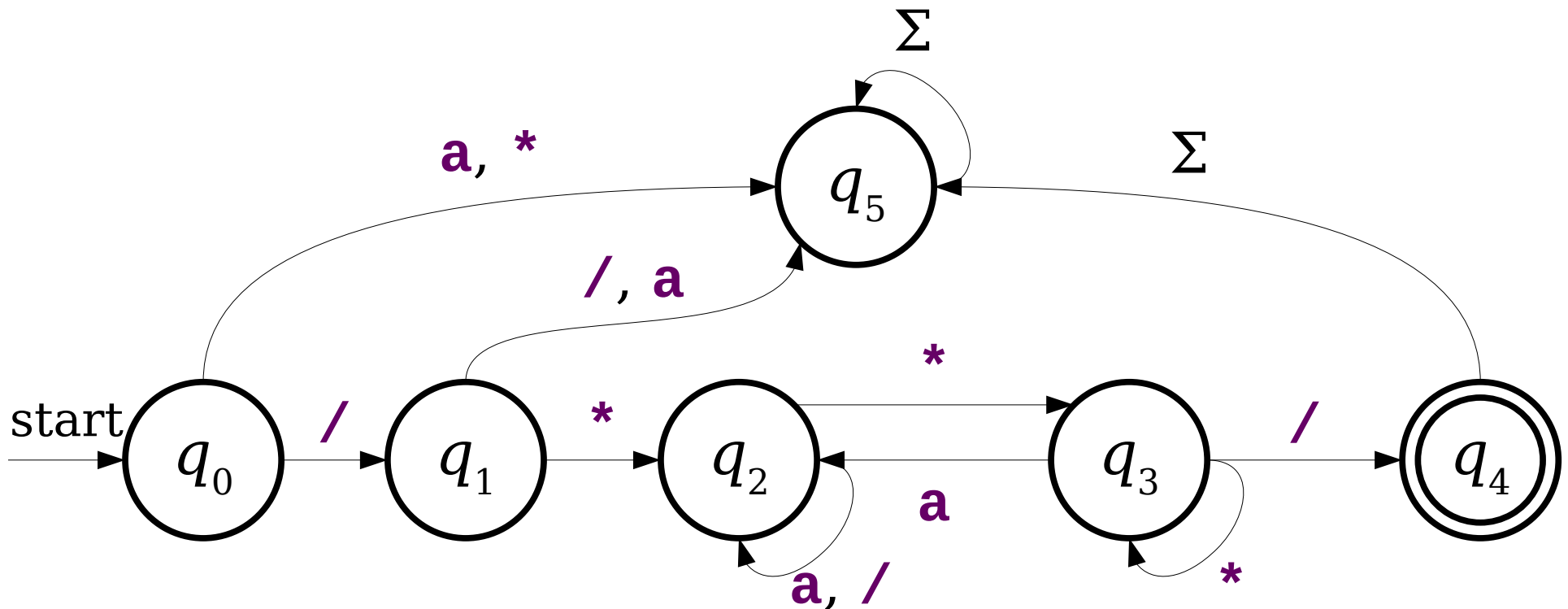
`/*a*/`  
`/**/`  
`/***/`  
`/*aaa*aaa*/`  
`/*a/a*/`

Rejected:

`/**`  
`/**/a/*aa*/`  
`aaa/**/aa`  
`/*/`  
`/**a/`  
`//aaaa`

# More Elaborate DFAs

$L = \{ w \in \{a, *, /\}^* \mid w \text{ represents a C-style comment} \}$

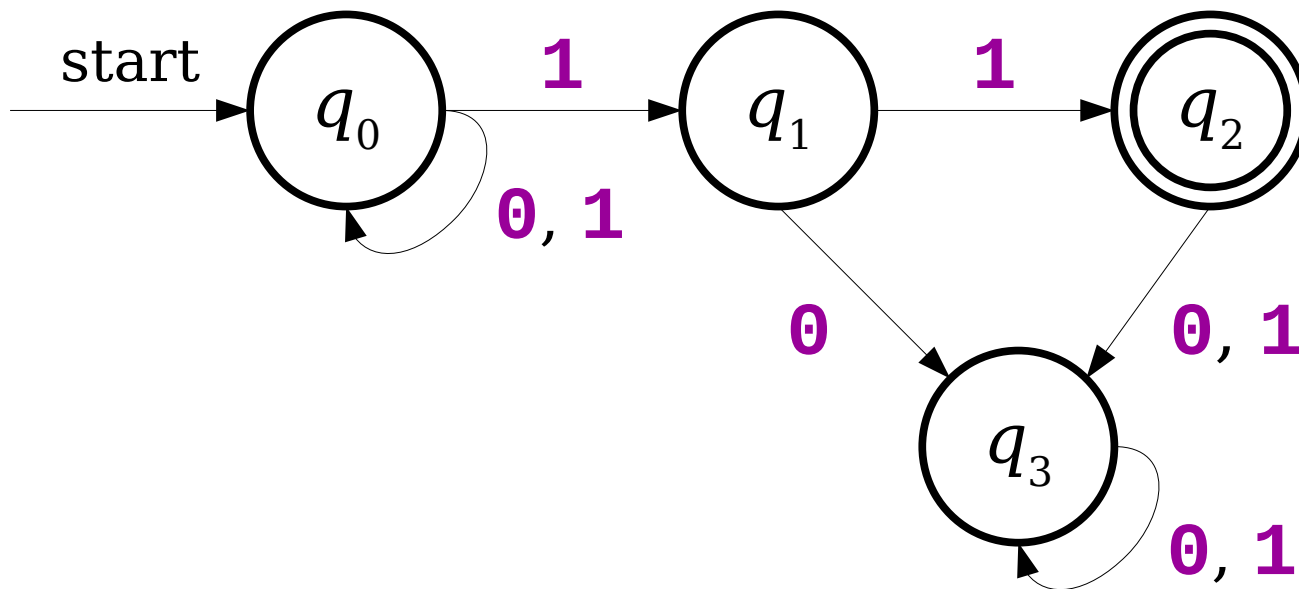


# The Regular Languages

A language  $L$  is called a ***regular language*** if there exists a DFA  $D$  such that  $\mathcal{L}(D) = L$ .

If  $L$  is a language and  $\mathcal{L}(D) = L$ , we say that  $D$  ***recognizes*** the language  $L$ .

# Revisiting a Problem



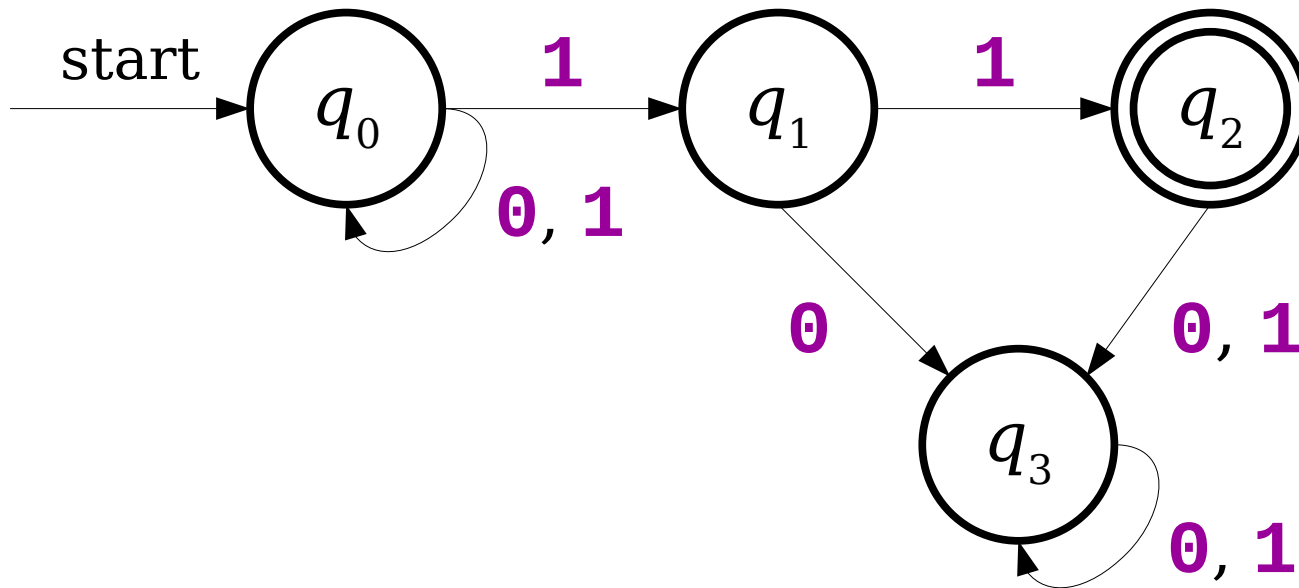
# NFAs

- An *NFA* is a
  - *N*ondeterministic
  - *F*inite
  - *A*utomaton
- Structurally similar to a DFA, but represents a fundamental shift in how we'll think about computation.

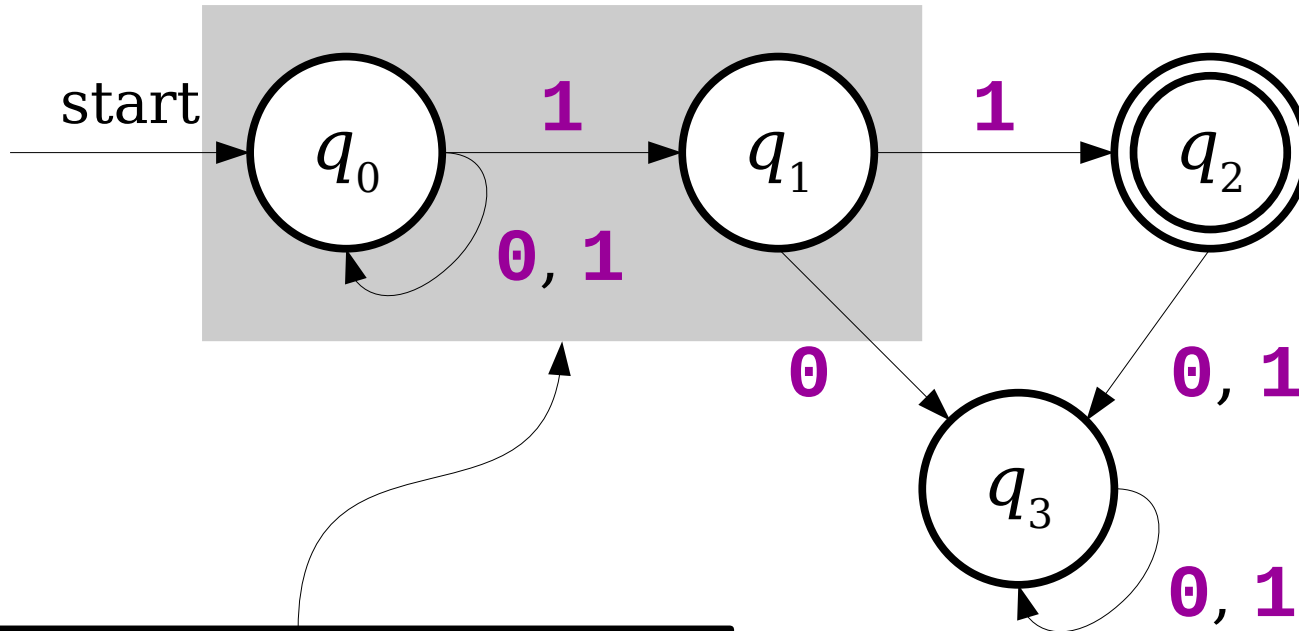
# (Non)determinism

- A model of computation is **deterministic** if at every point in the computation, there is exactly one choice that can make.
  - The machine accepts if that series of choices leads to an accepting state.
- A model of computation is **nondeterministic** if the computing machine has a finite number of choices available to make at each point, possibly including zero.
- The machine accepts if **any** series of choices leads to an accepting state.
  - (This sort of nondeterminism is technically called **existential nondeterminism**, the most philosophical-sounding term we'll introduce all quarter.)

# A Simple NFA

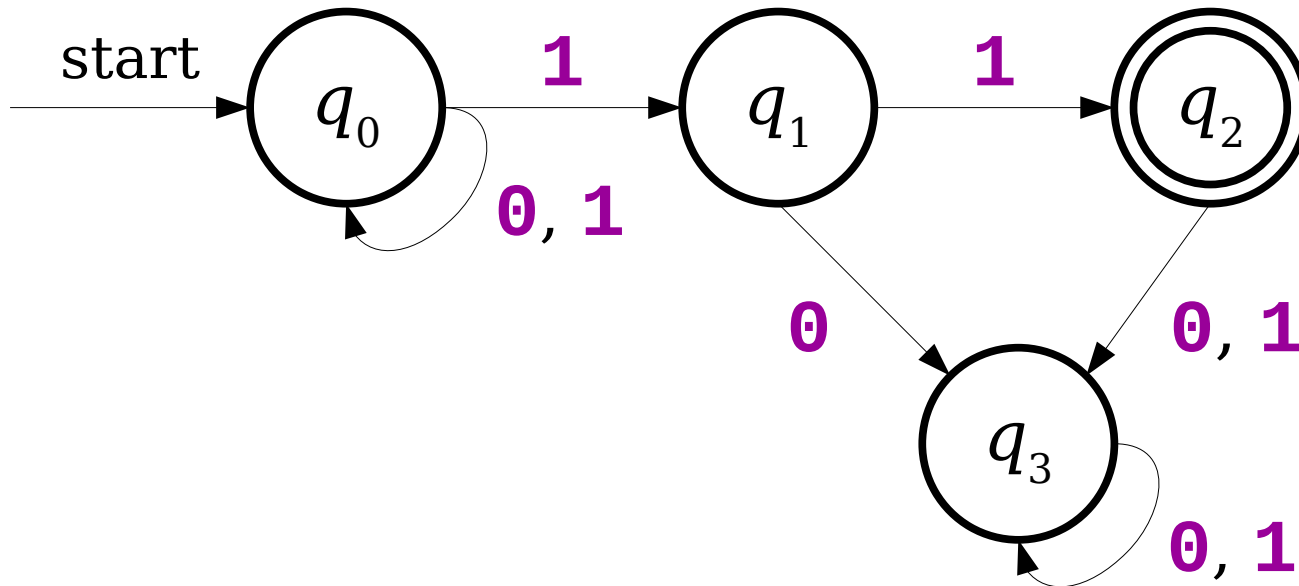


# A Simple NFA



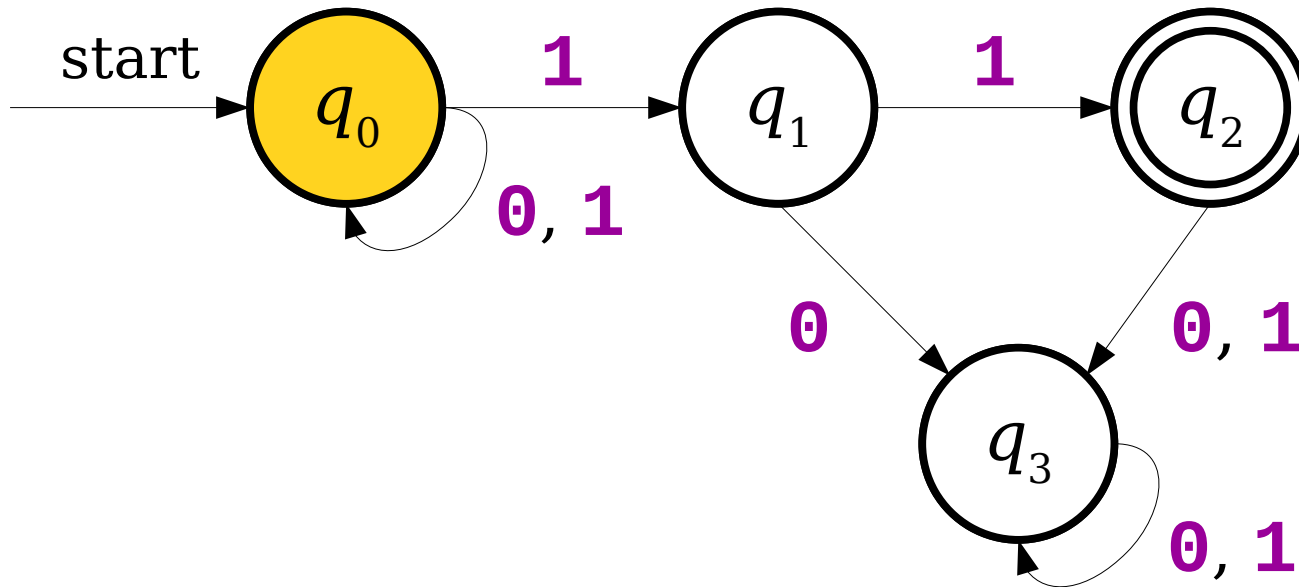
$q_0$  has two transitions  
defined on 1!

# A Simple NFA



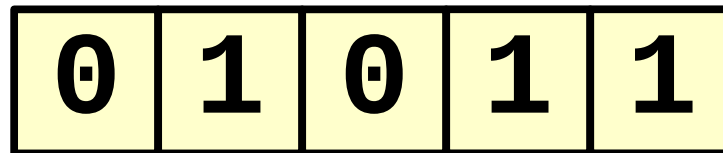
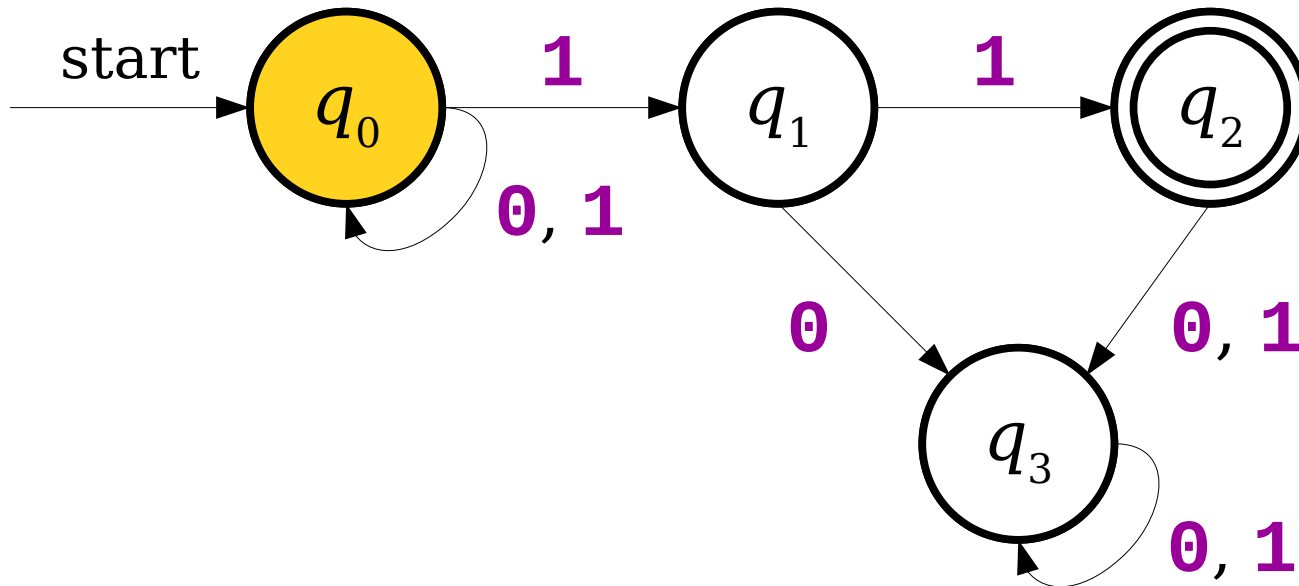
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# A Simple NFA

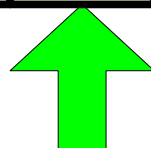
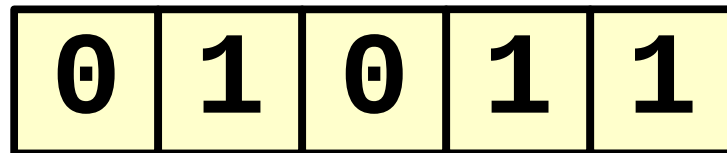
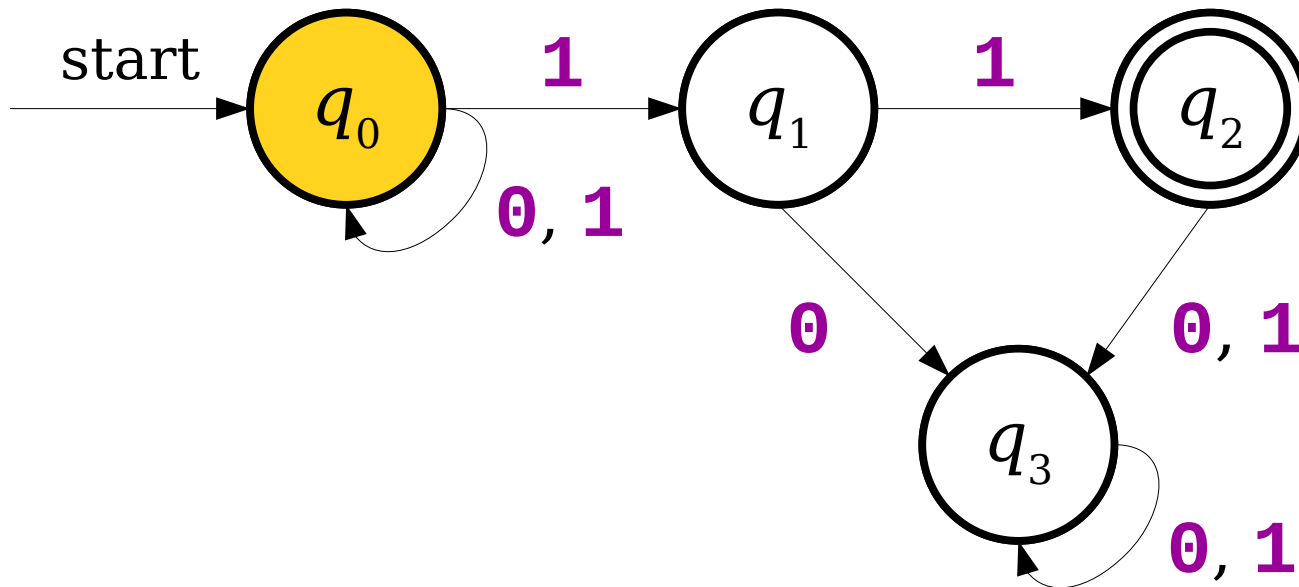


<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>1</b>
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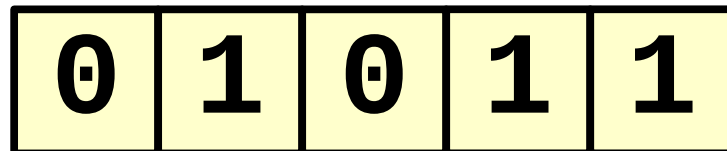
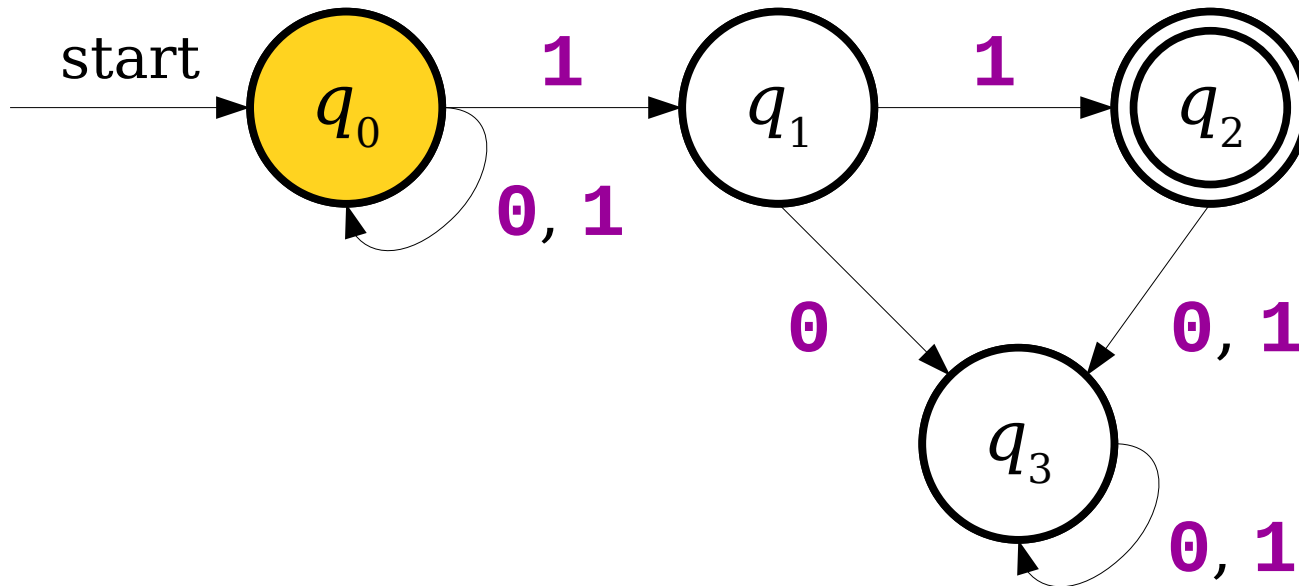
# A Simple NFA



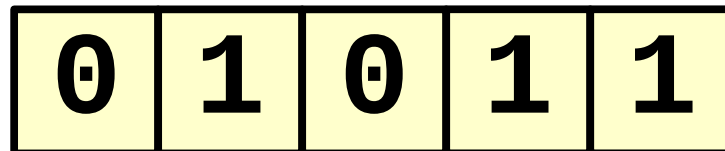
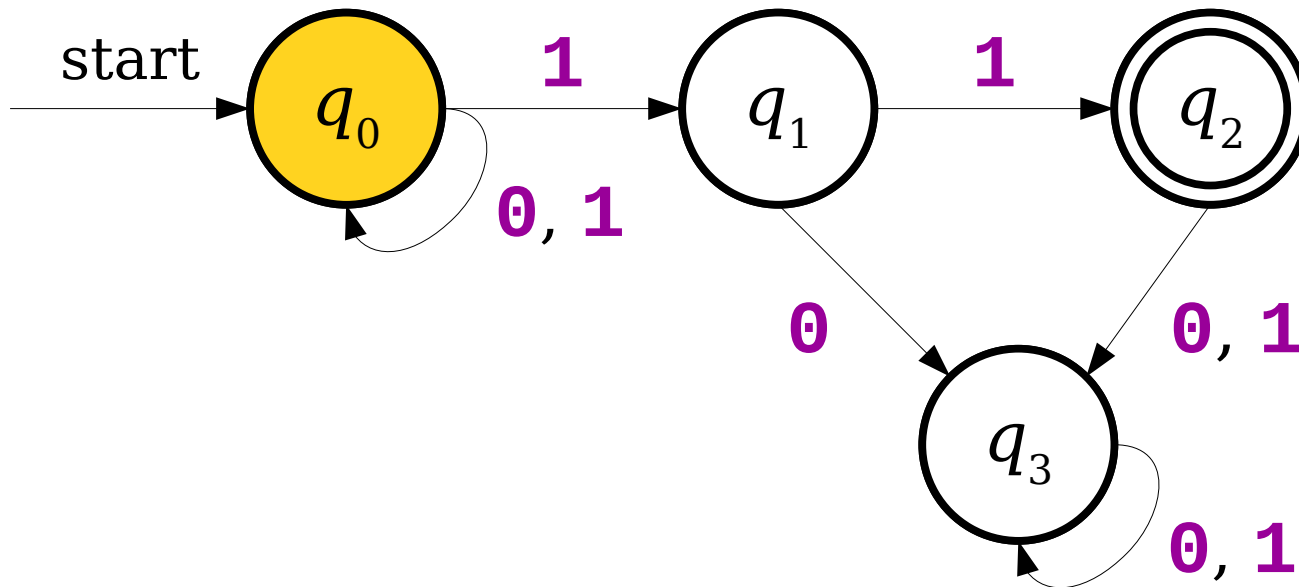
# A Simple NFA



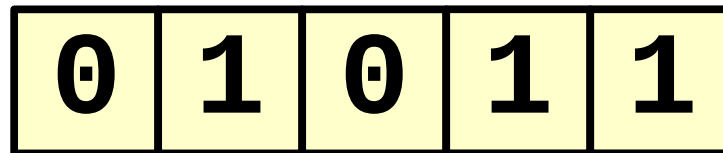
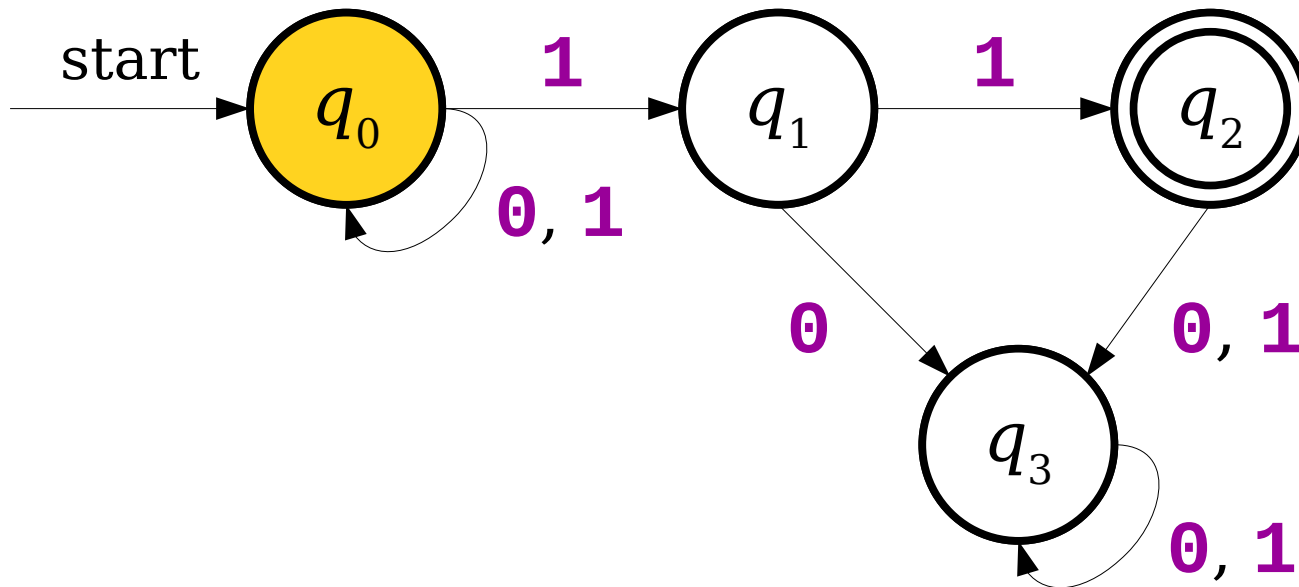
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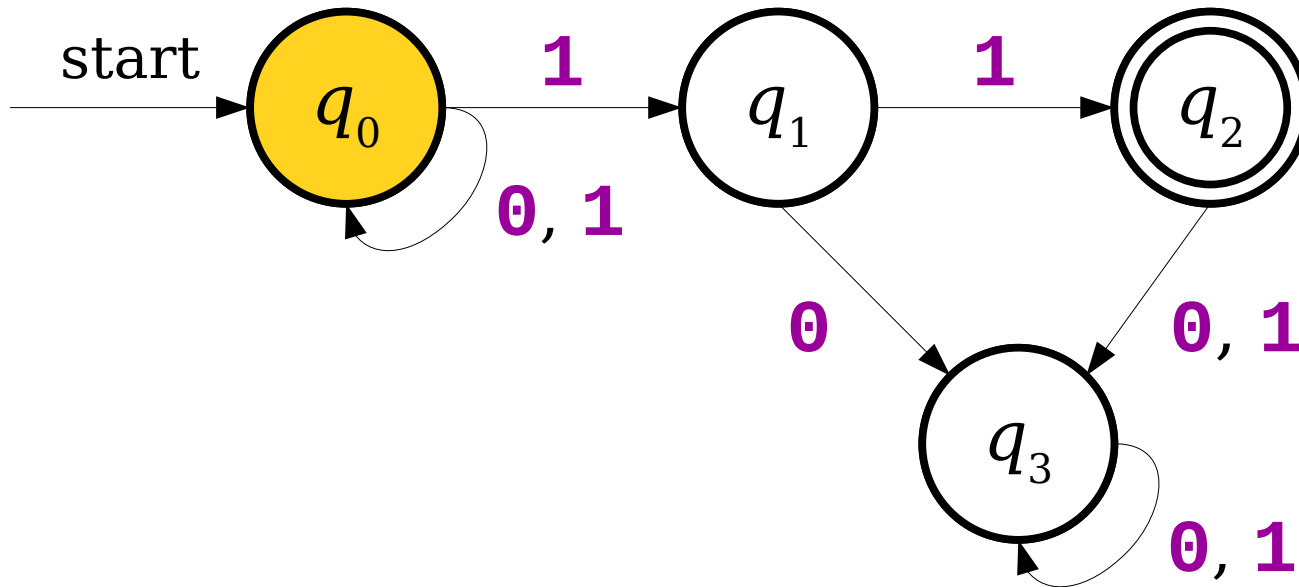
# A Simple NFA



# A Simple NFA

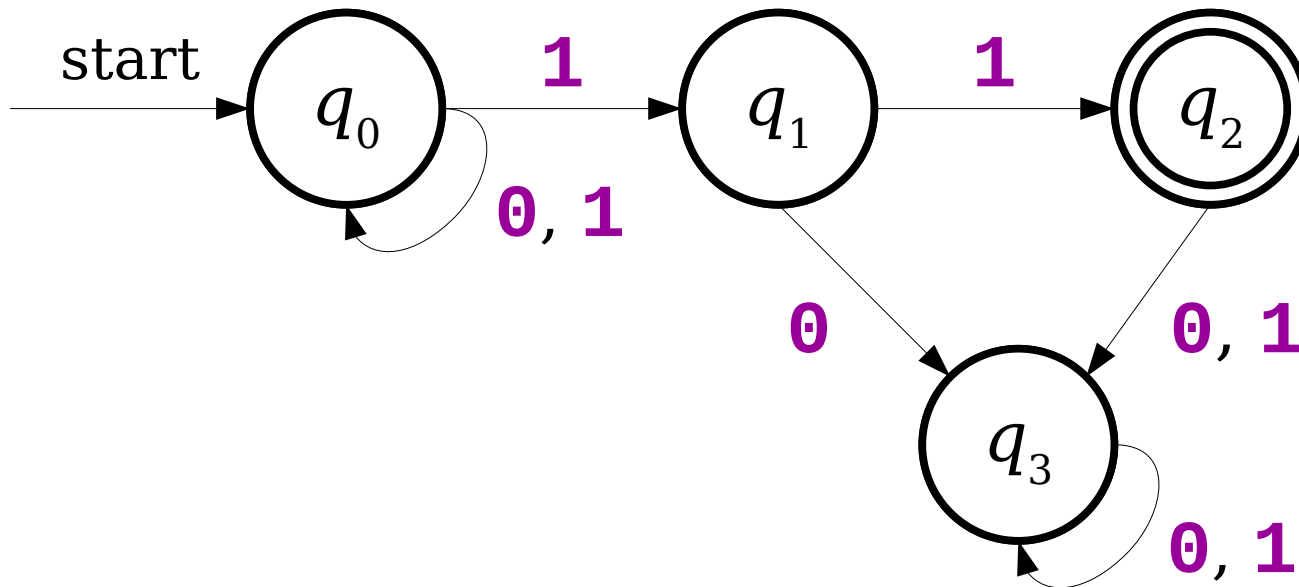


# A Simple NFA



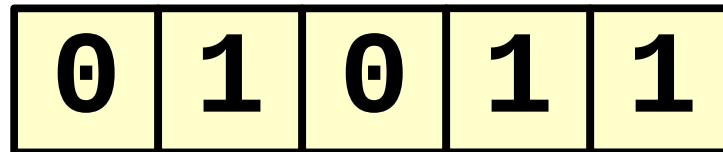
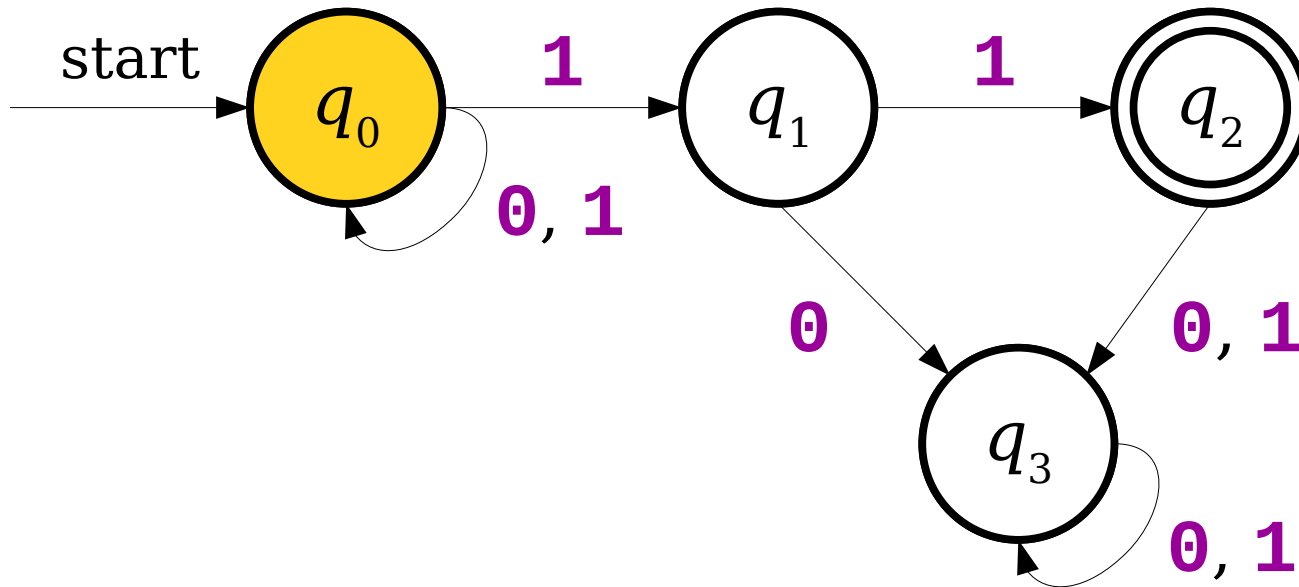
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# A Simple NFA

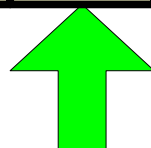
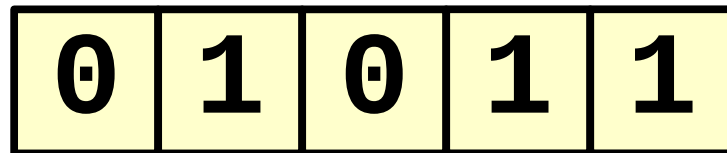
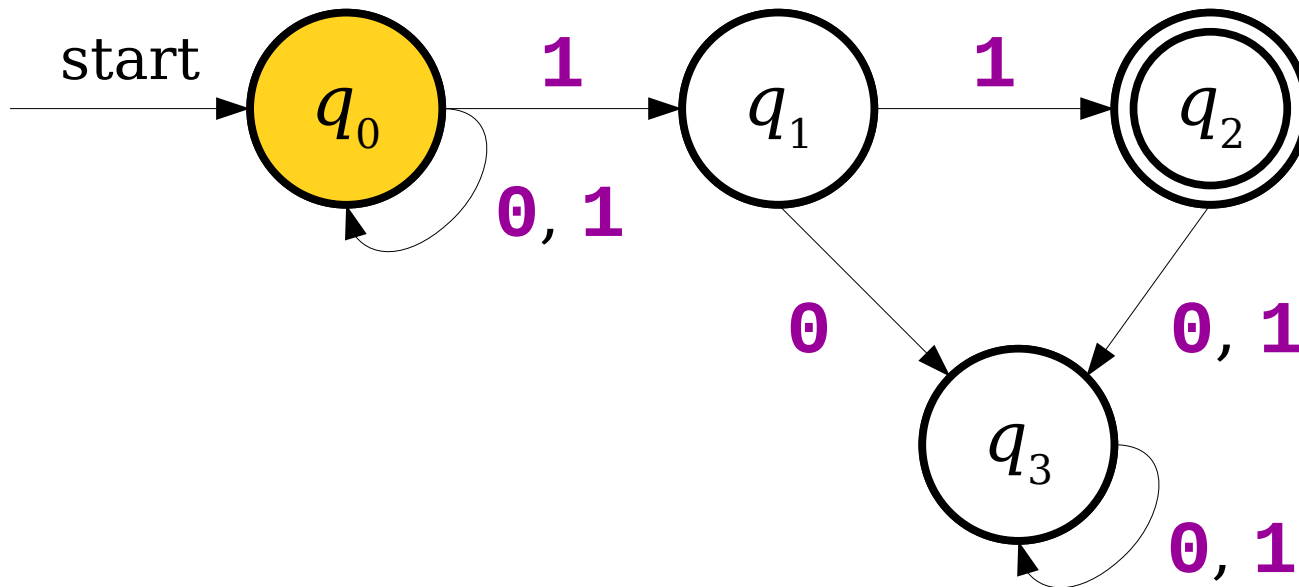


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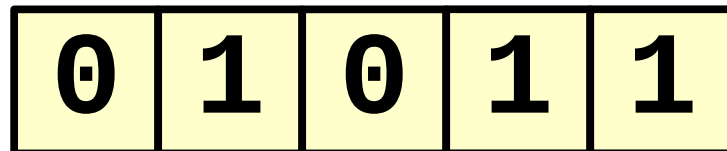
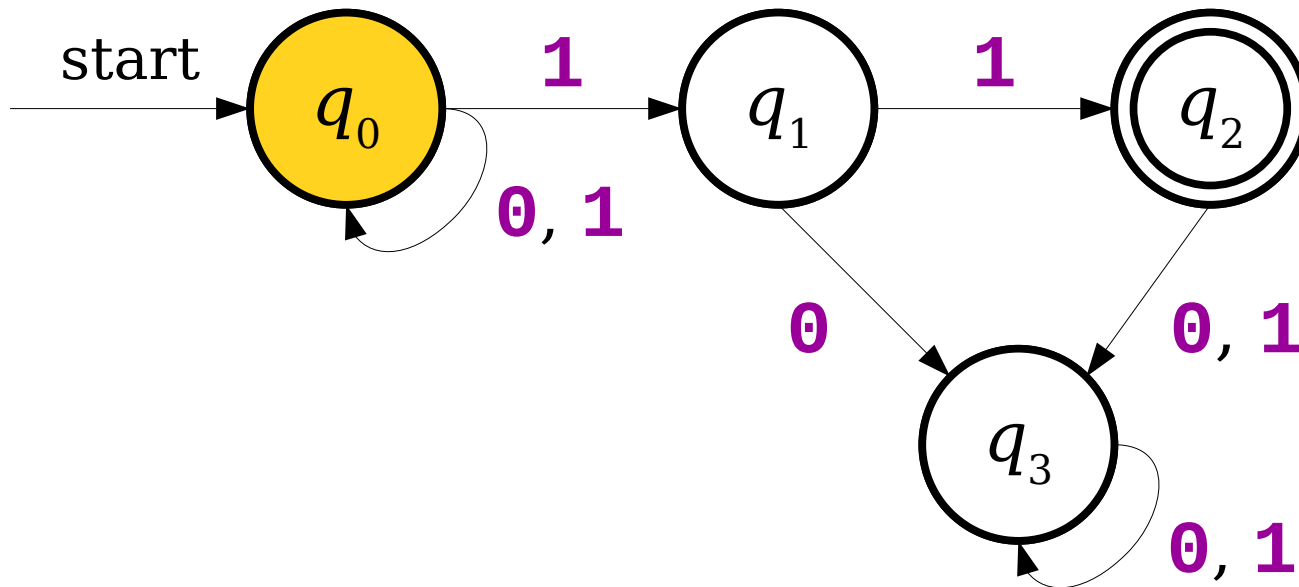
# A Simple NFA



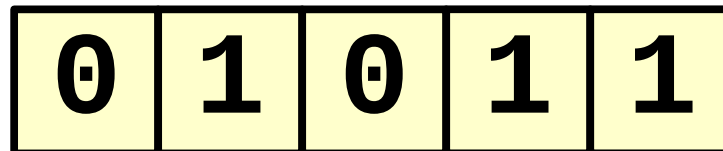
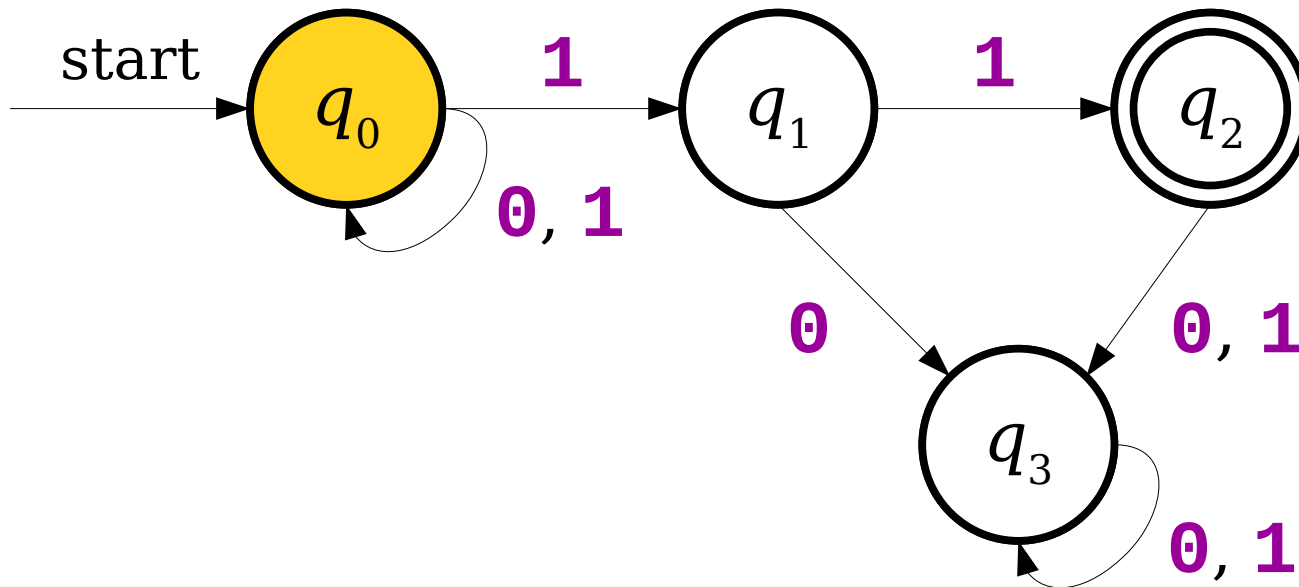
# A Simple NFA



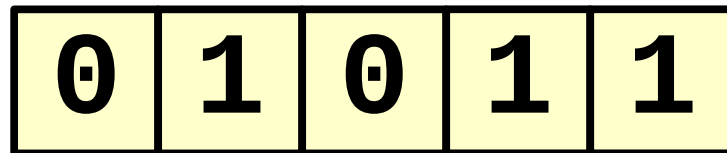
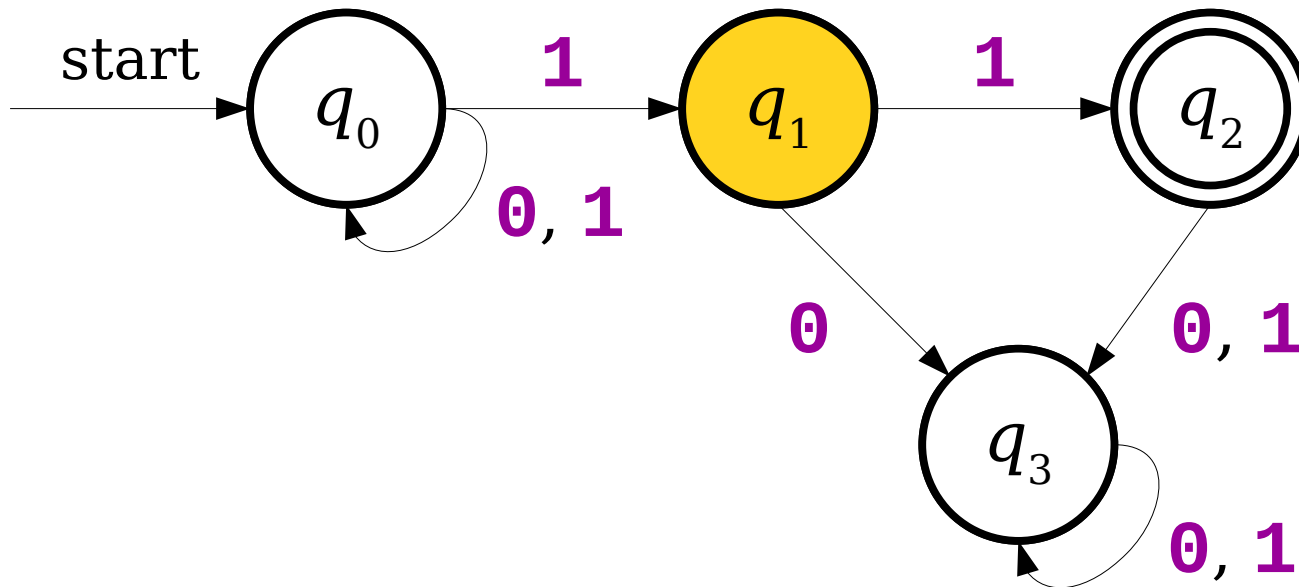
# A Simple NFA



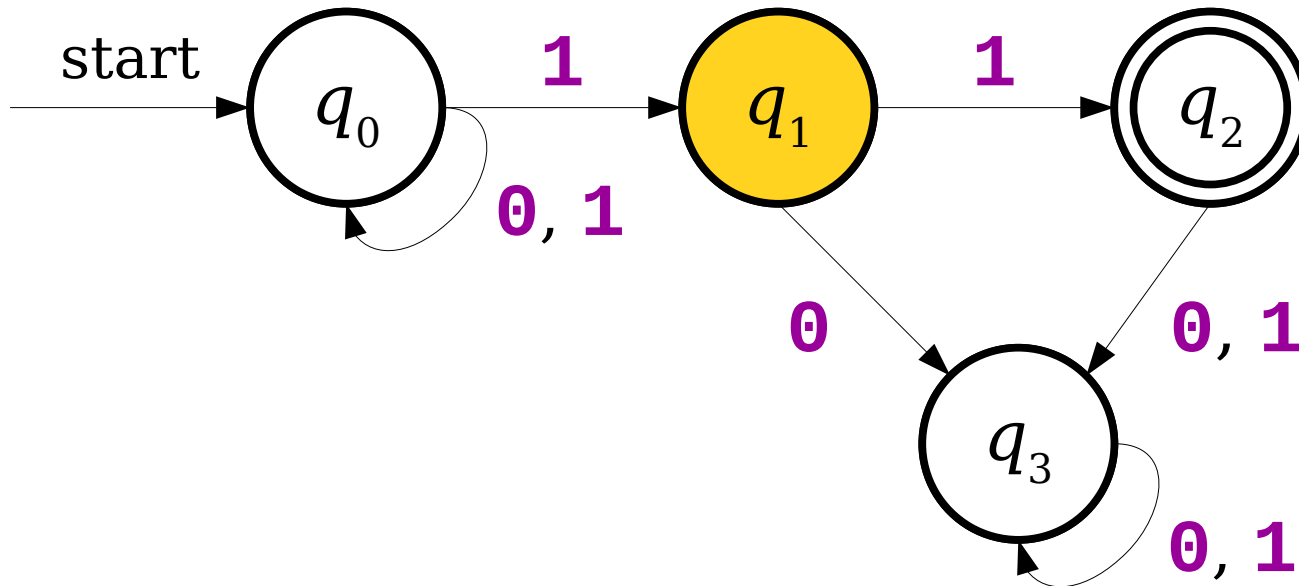
# A Simple NFA



# A Simple NFA



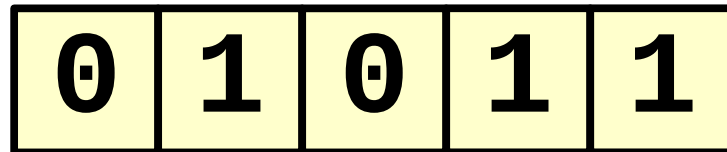
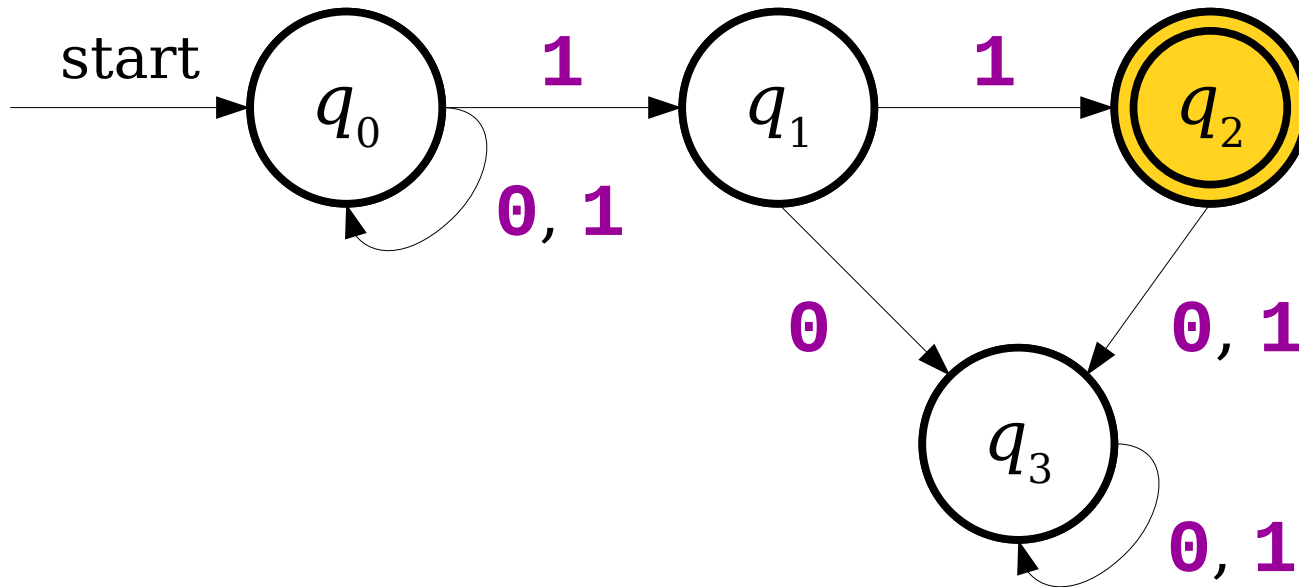
# A Simple NFA



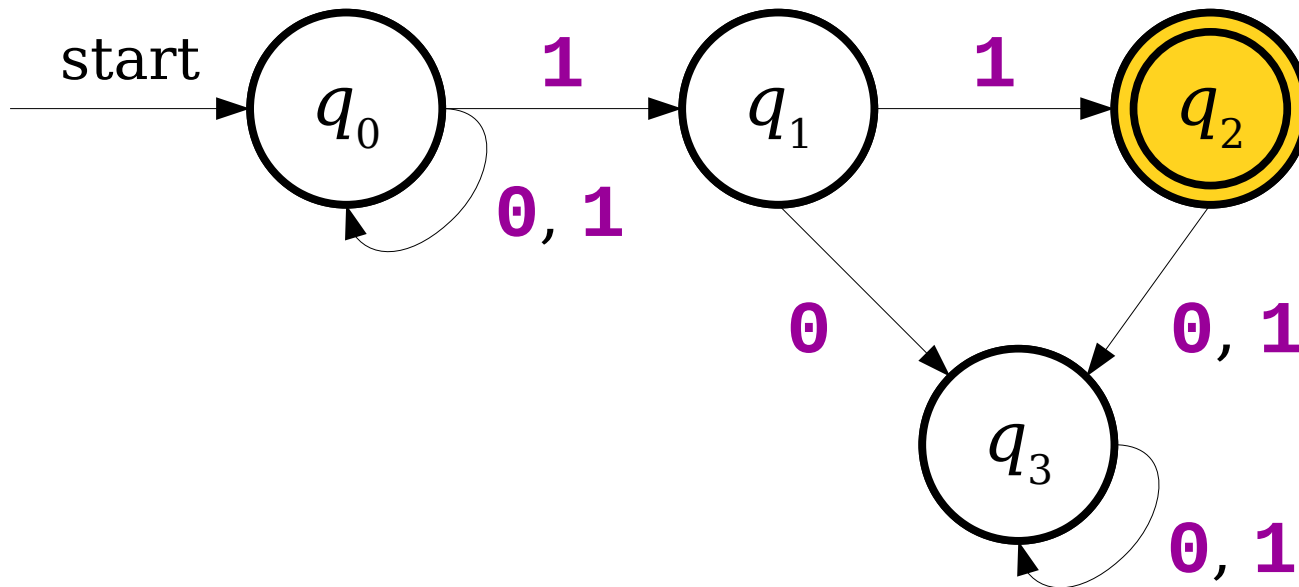
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# A Simple NFA

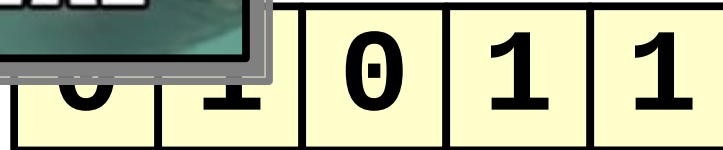
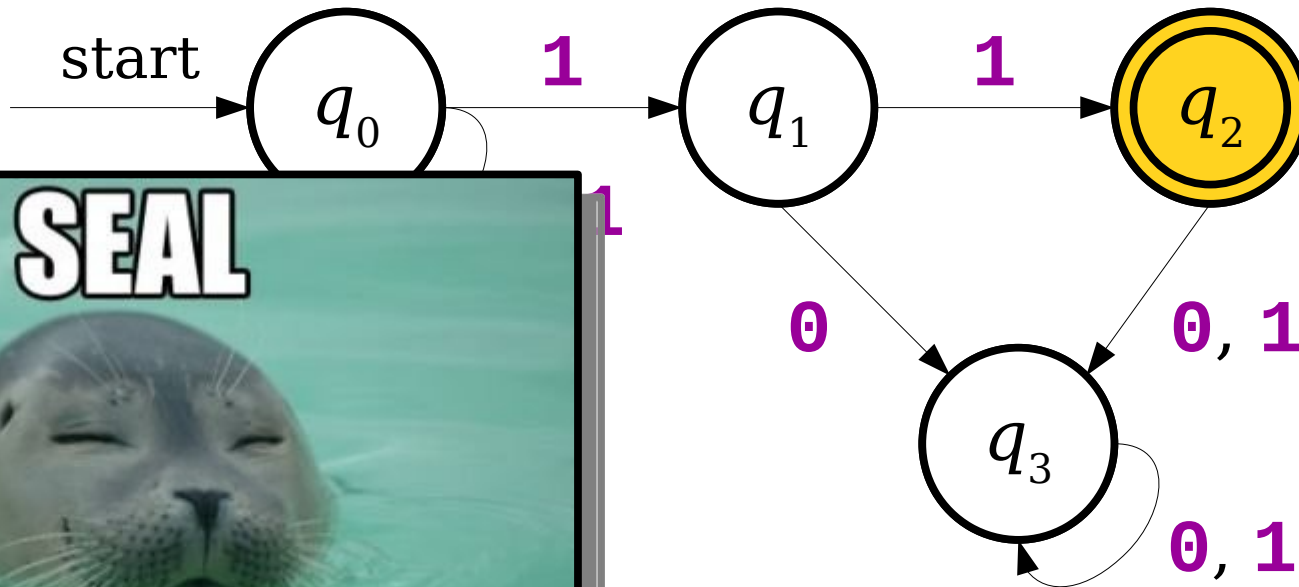


# A Simple NFA

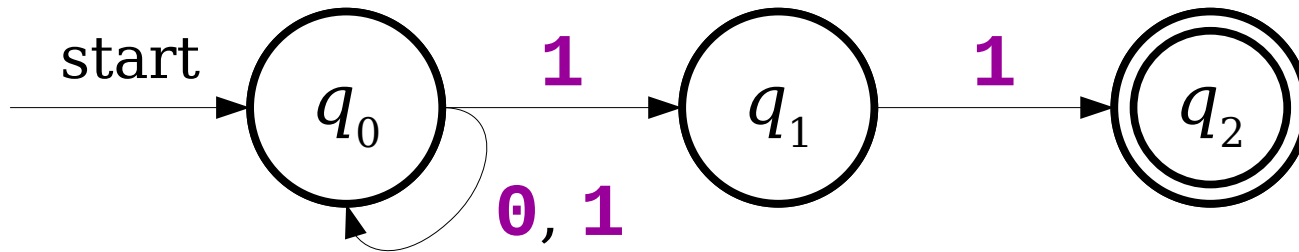


0	1	0	1	1
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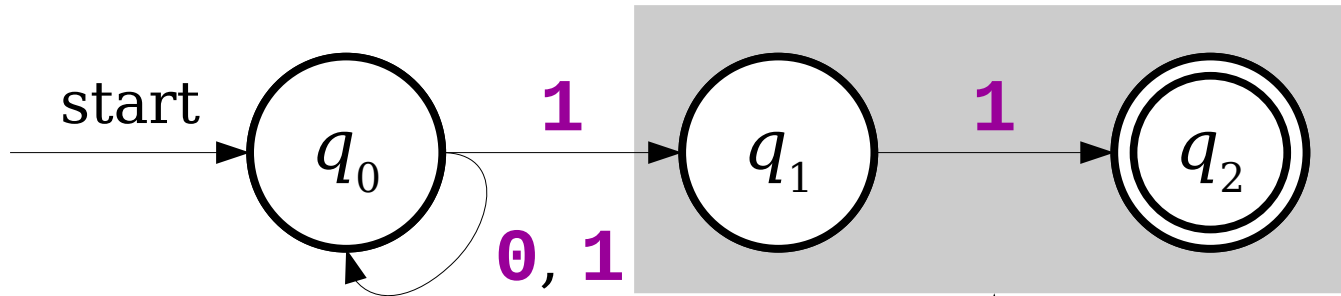
# A Simple NFA



# A More Complex NFA

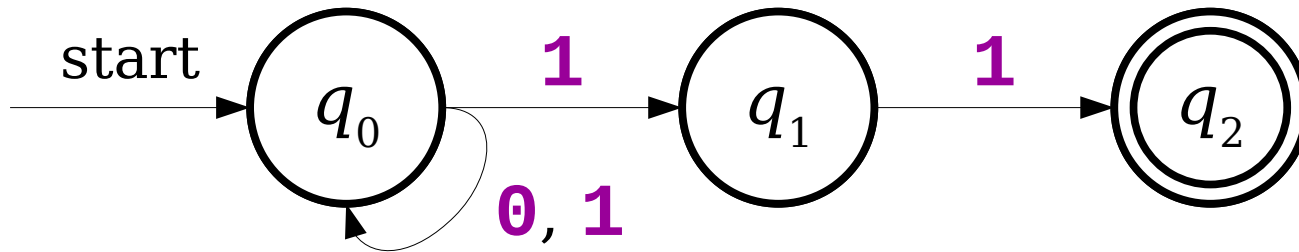


# A More Complex NFA



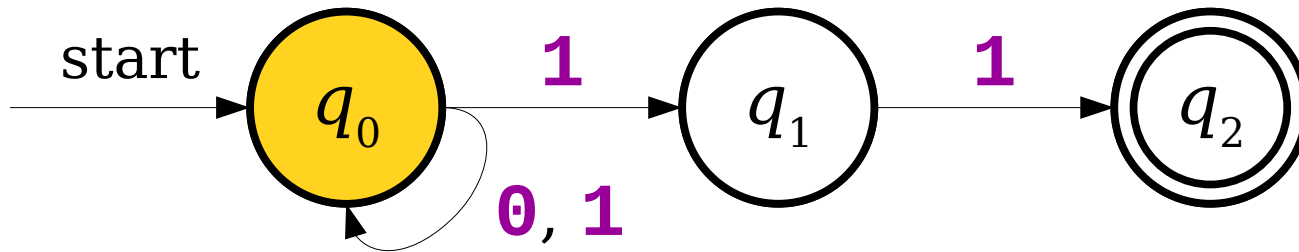
If a NFA needs to make a transition when no transition exists, the automaton **dies** and that particular path does not accept.

# A More Complex NFA



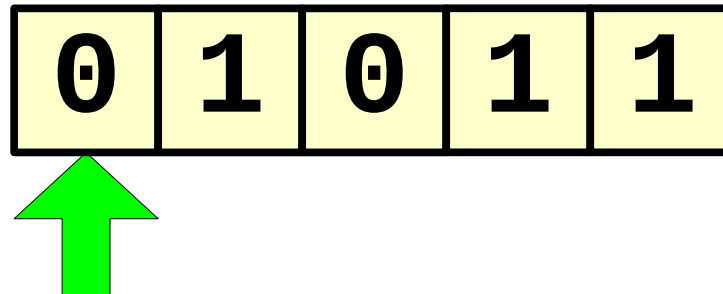
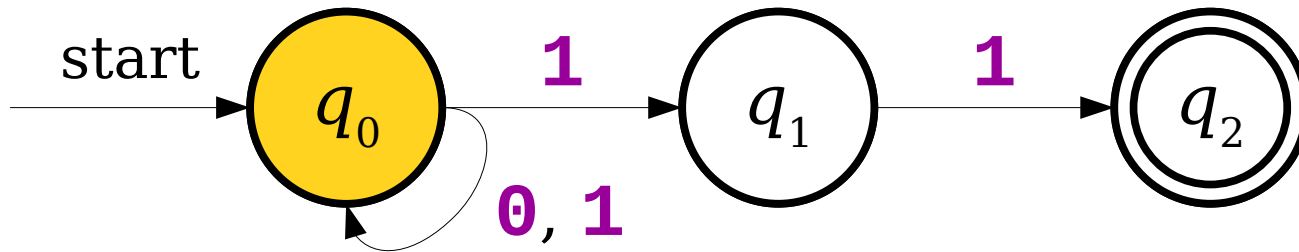
0	1	0	1	1
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# A More Complex NFA

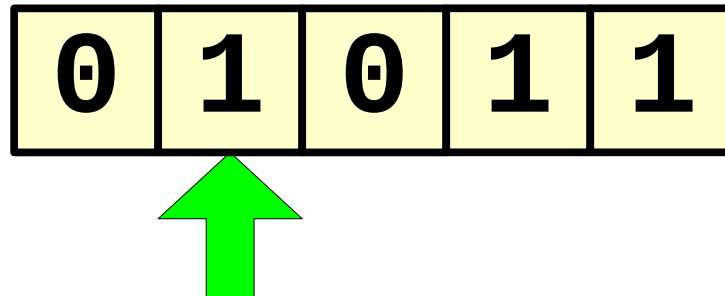
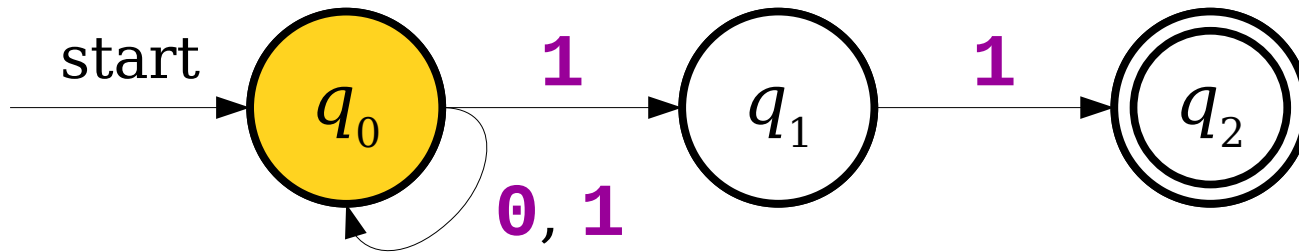


0	1	0	1	1
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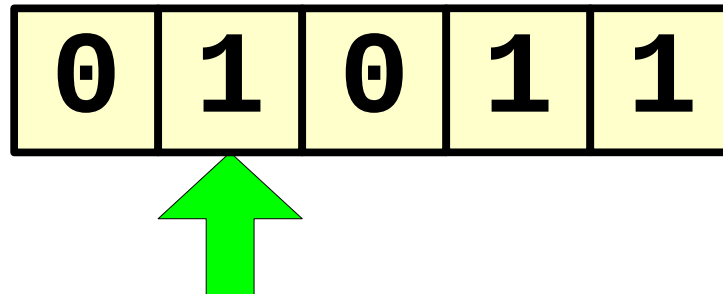
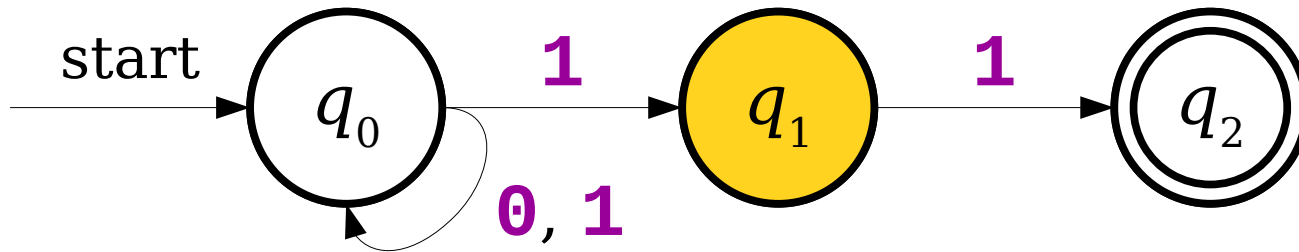
# A More Complex NFA



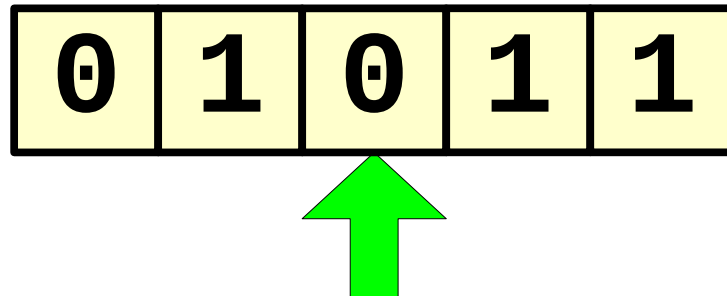
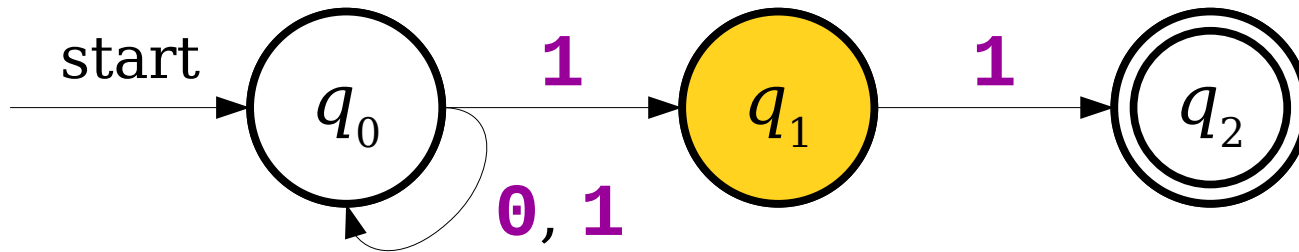
# A More Complex NFA



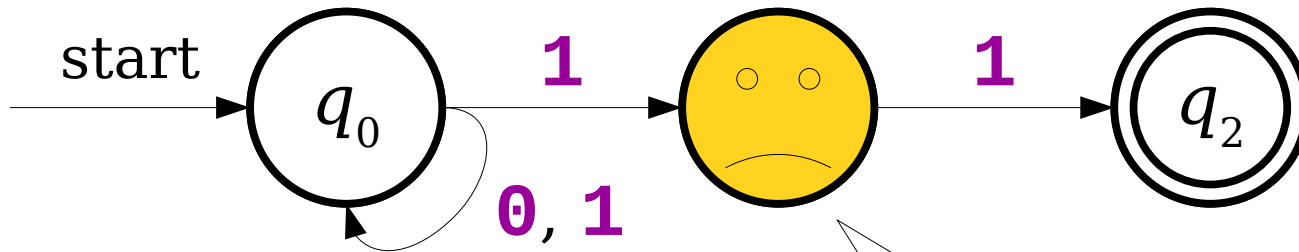
# A More Complex NFA



# A More Complex NFA

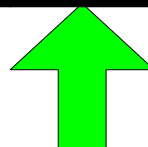


# A More Complex NFA

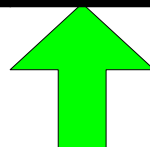
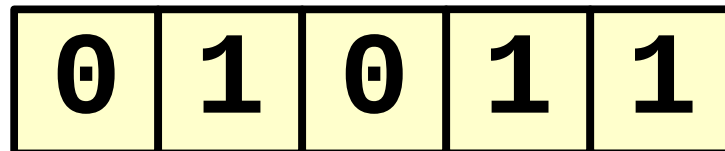
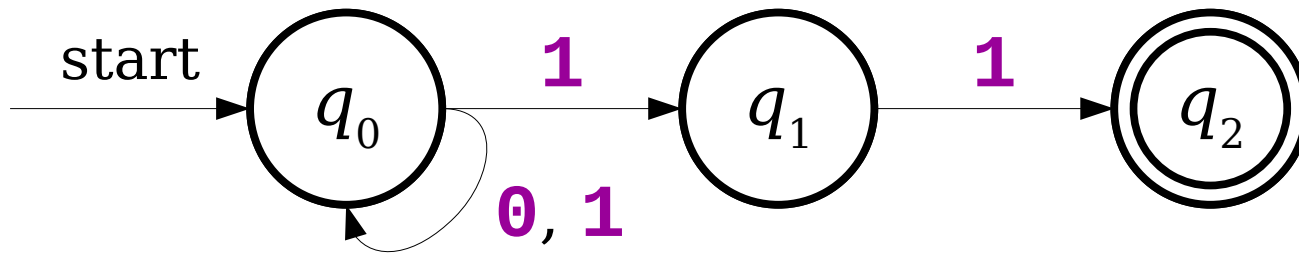


Oh no! There's no transition defined!

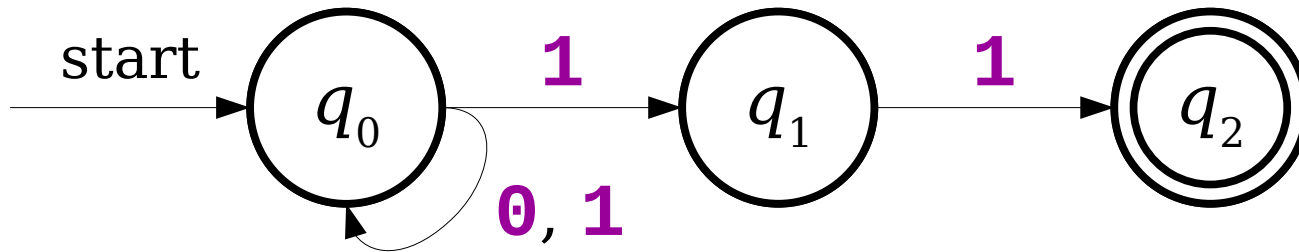
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# A More Complex NFA

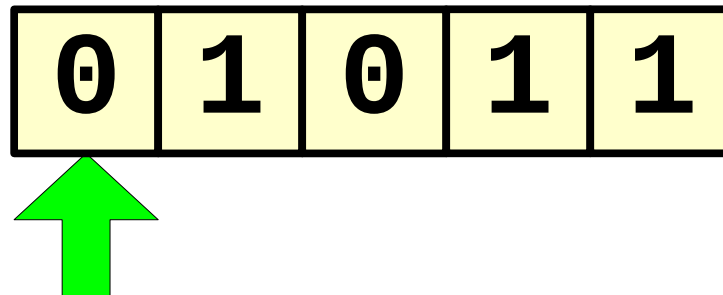
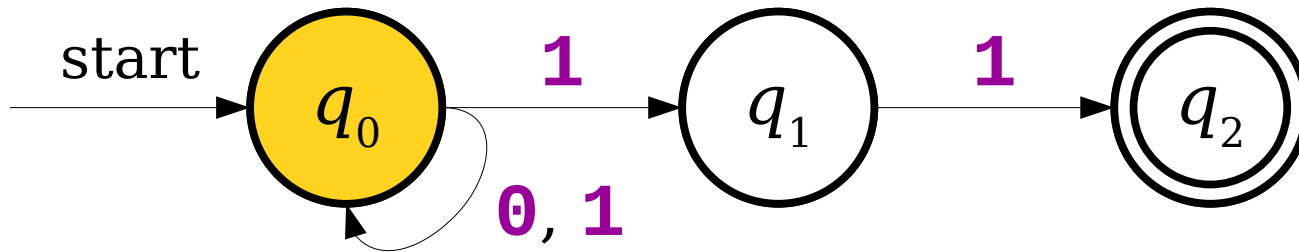


# A More Complex NFA

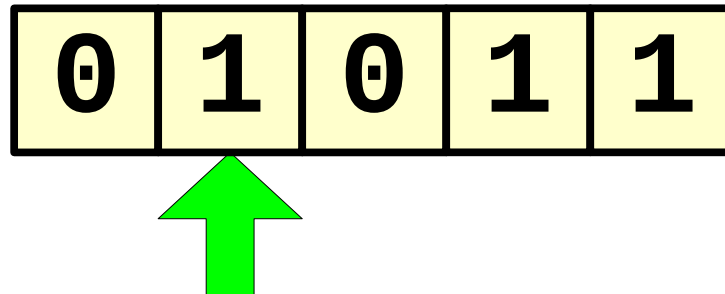
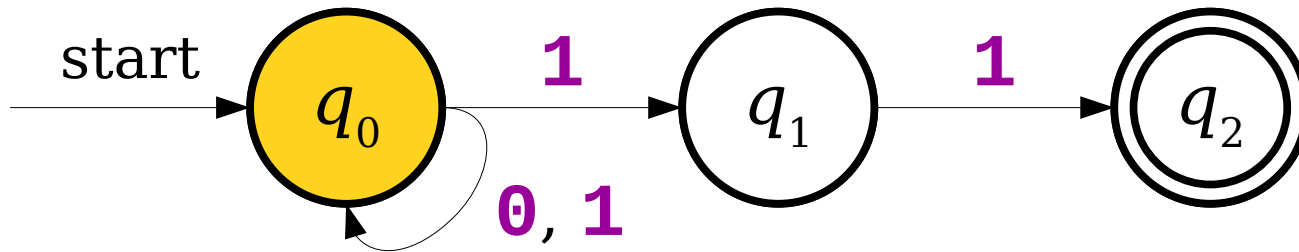


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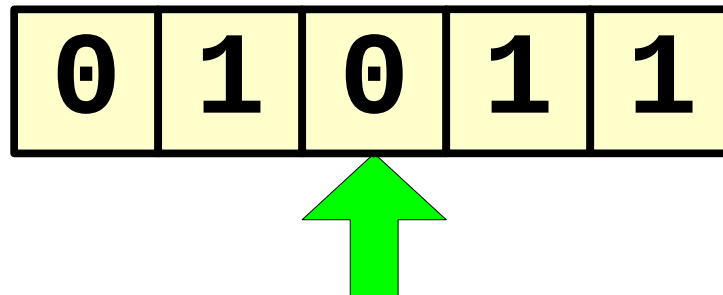
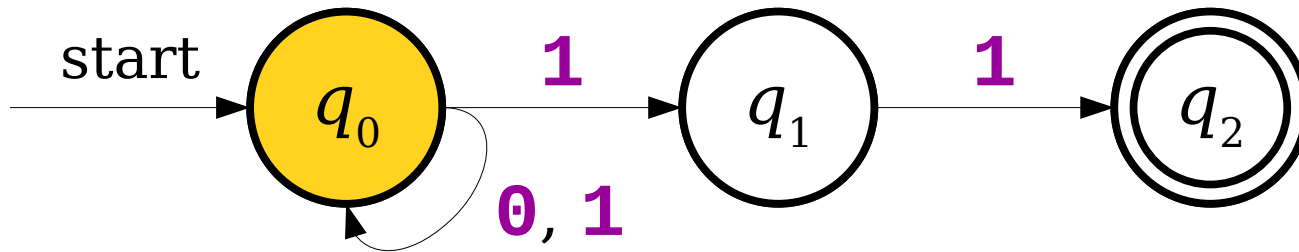
# A More Complex NFA



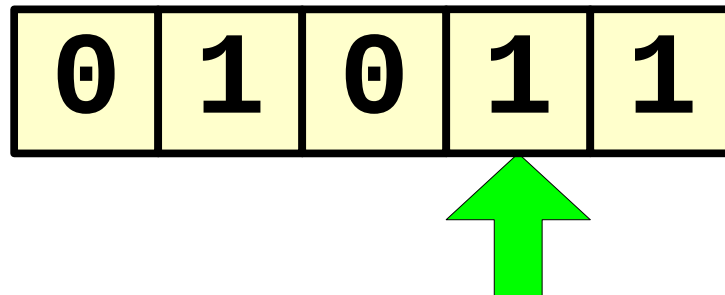
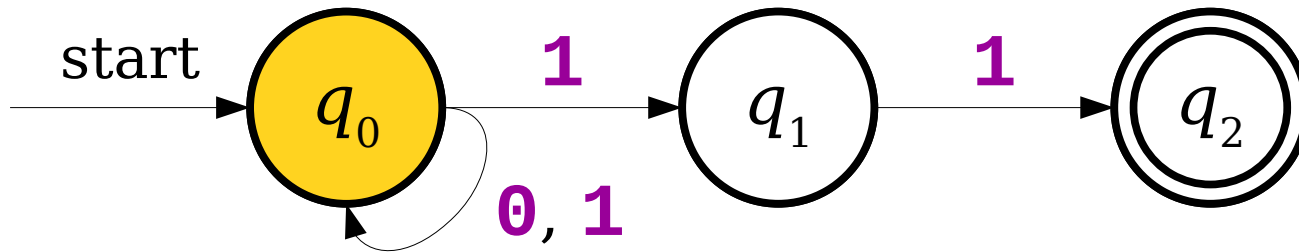
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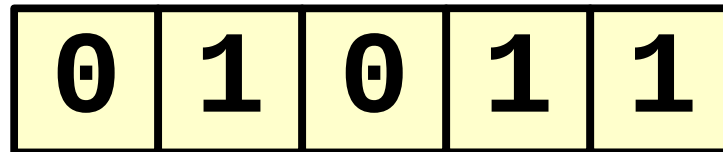
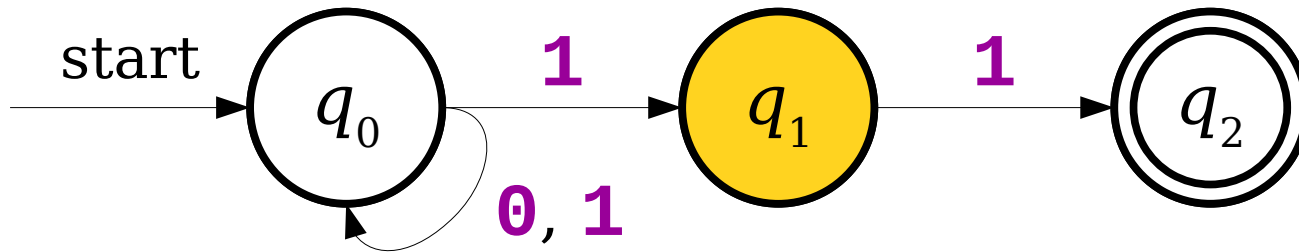
# A More Complex NFA



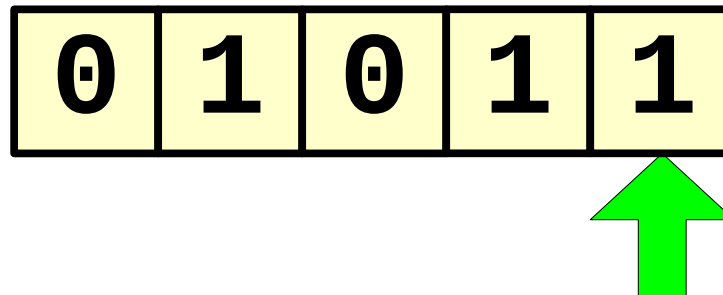
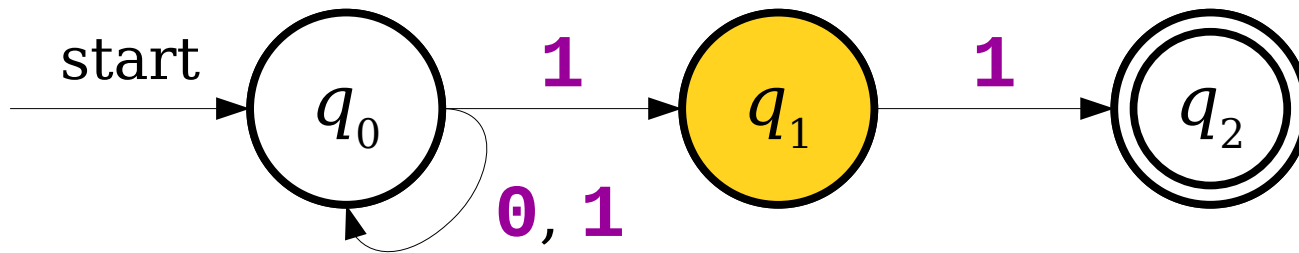
# A More Complex NFA



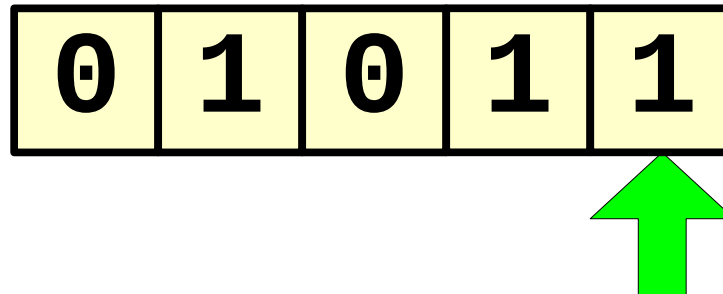
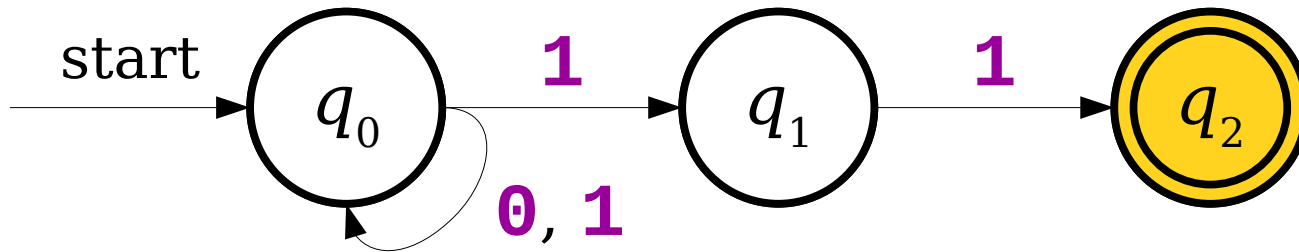
# A More Complex NFA



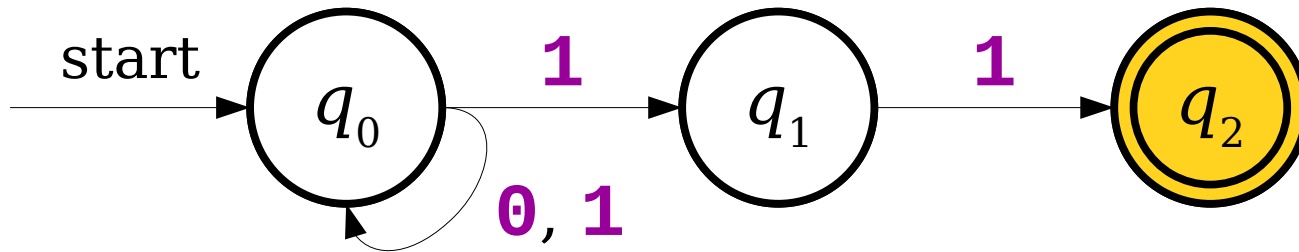
# A More Complex NFA



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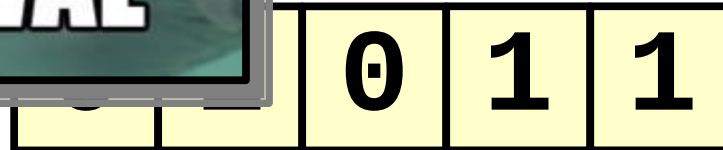
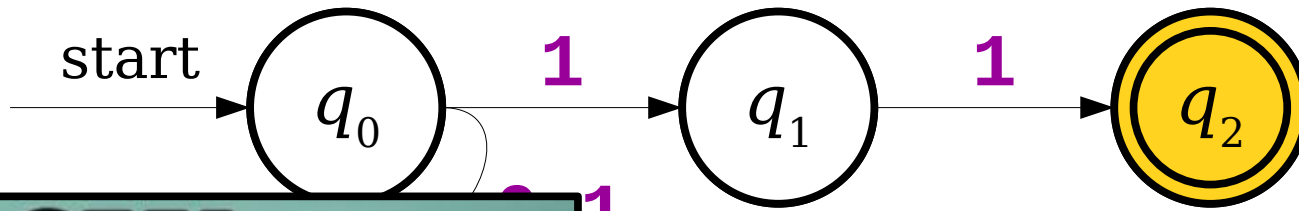


# A More Complex NFA

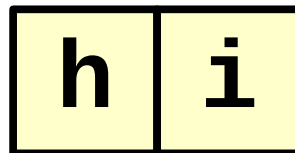
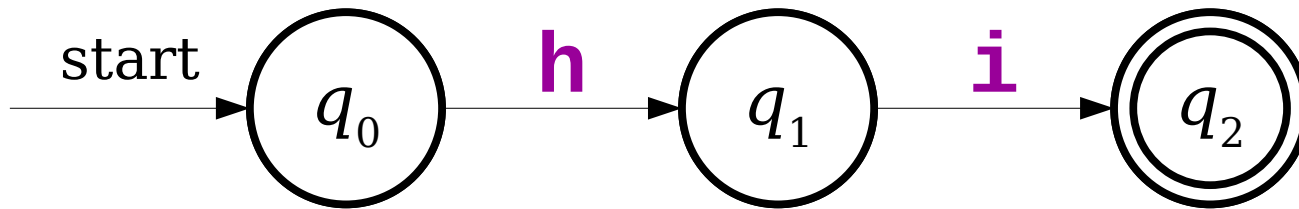


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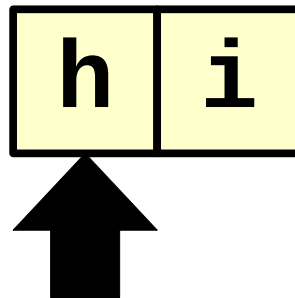
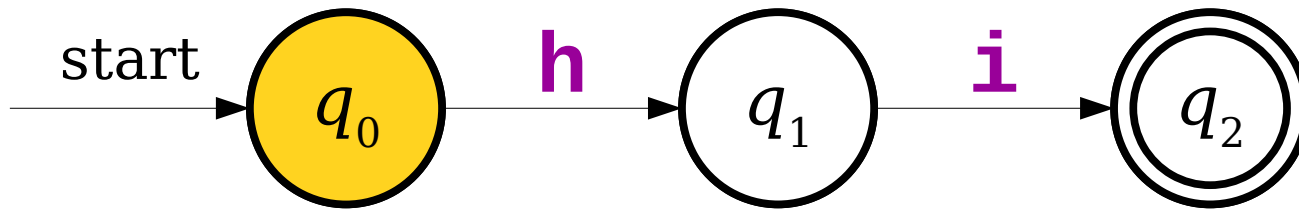
# A More Complex NFA



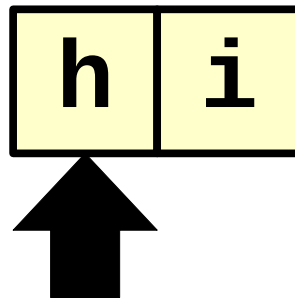
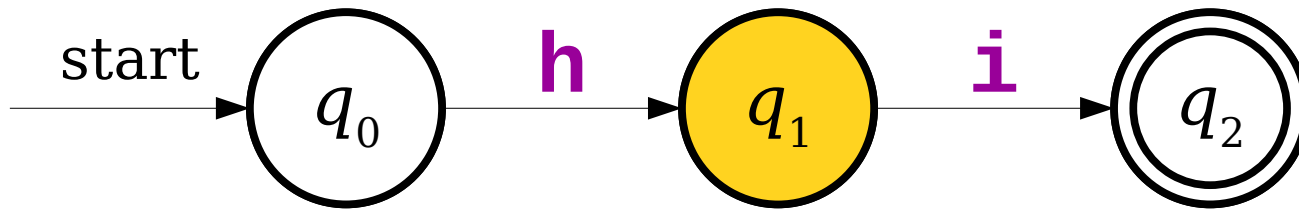
# Hello, NFA!



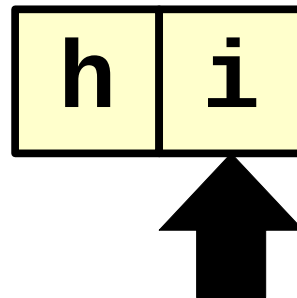
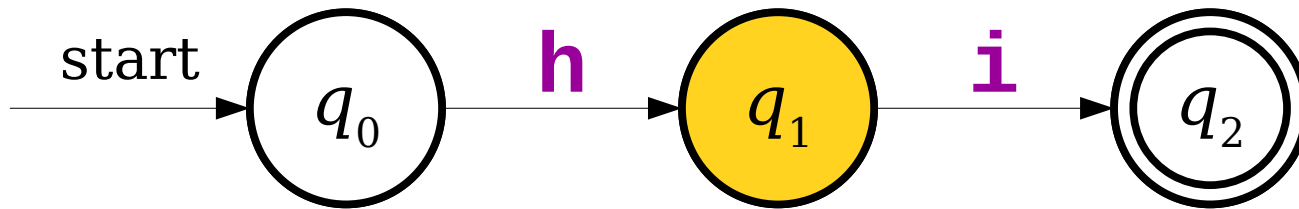
# Hello, NFA!



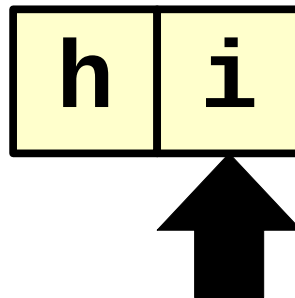
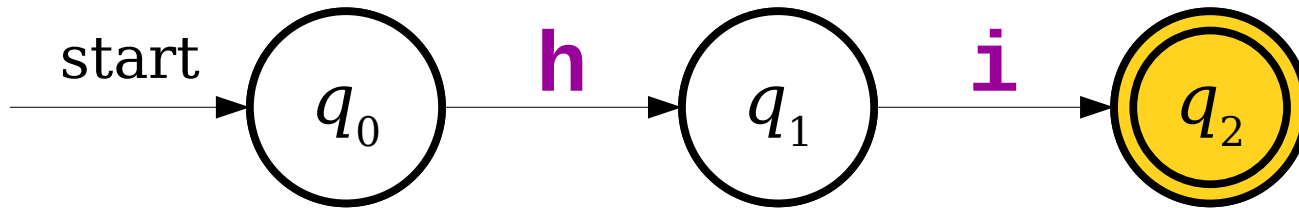
# Hello, NFA!



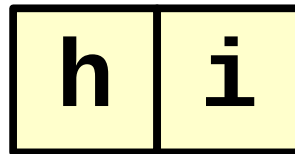
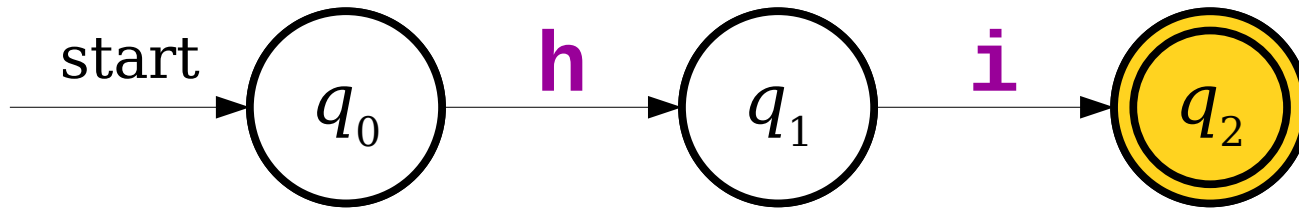
# Hello, NFA!



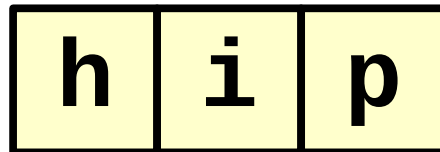
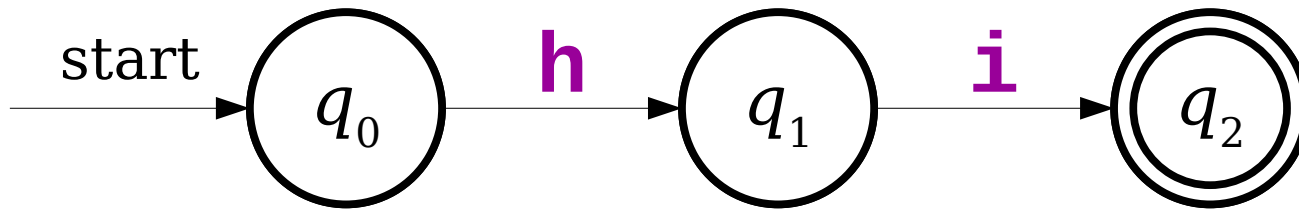
# Hello, NFA!



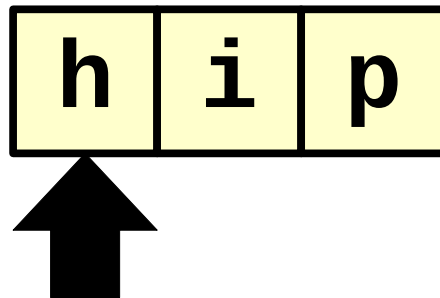
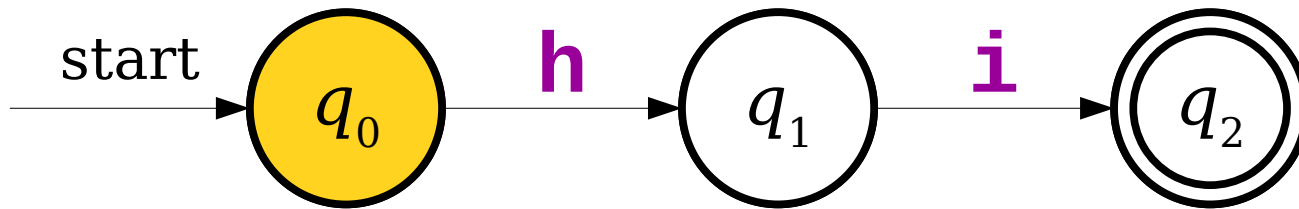
# Hello, NFA!



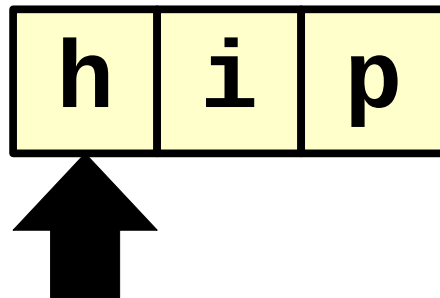
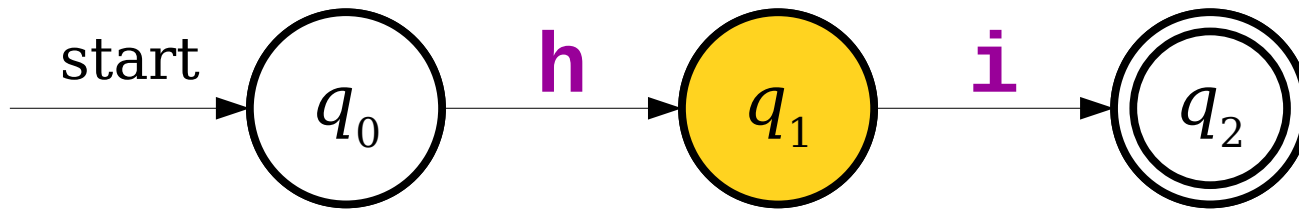
# Tragedy in Paradise



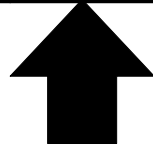
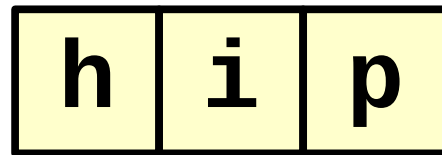
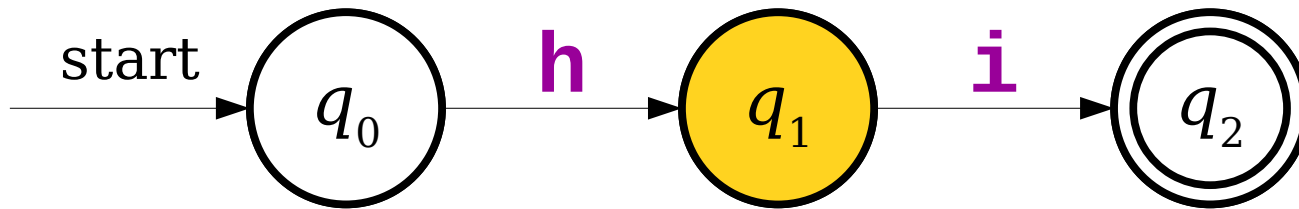
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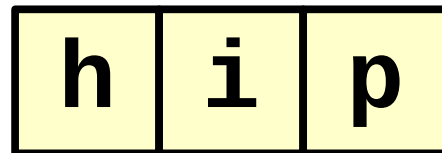
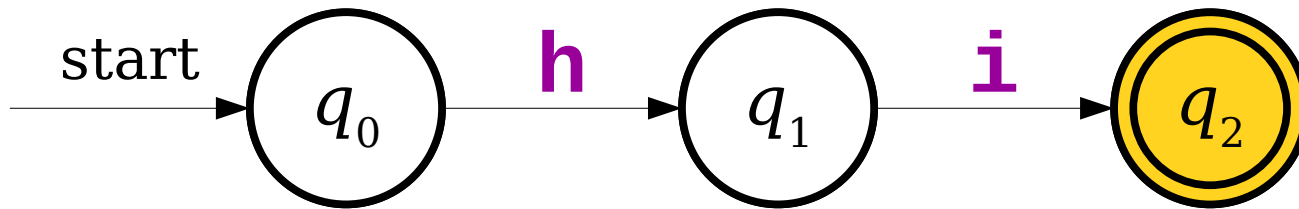
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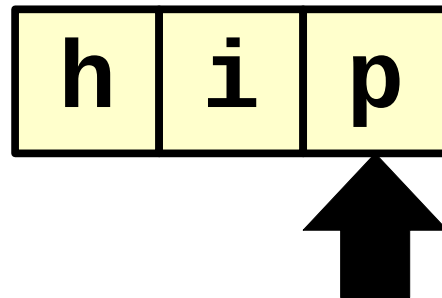
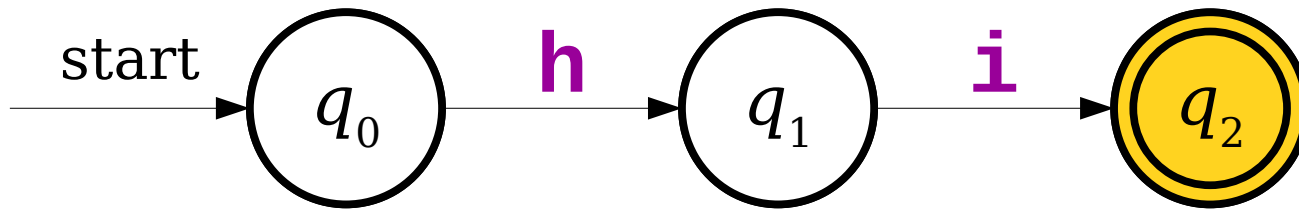
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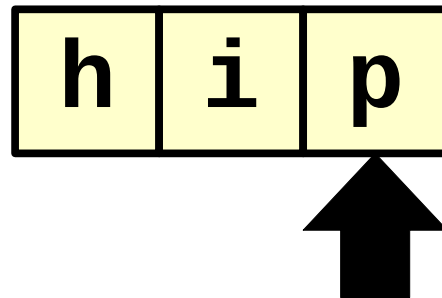
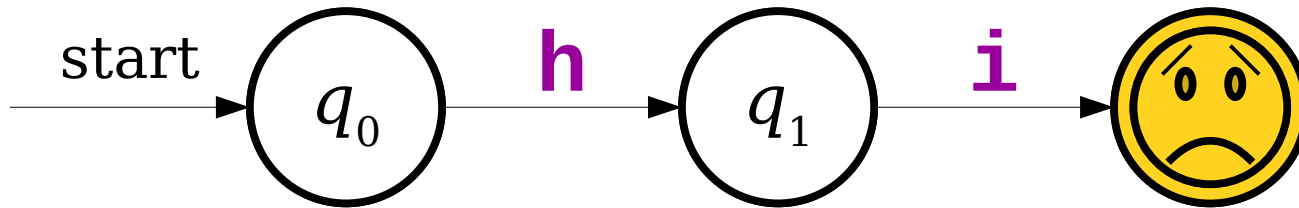
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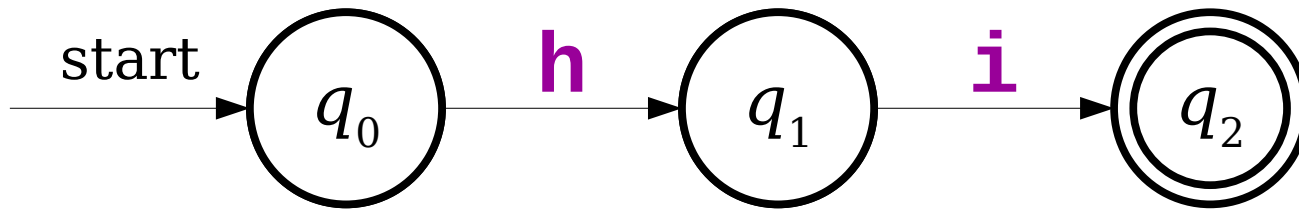
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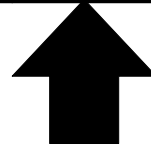
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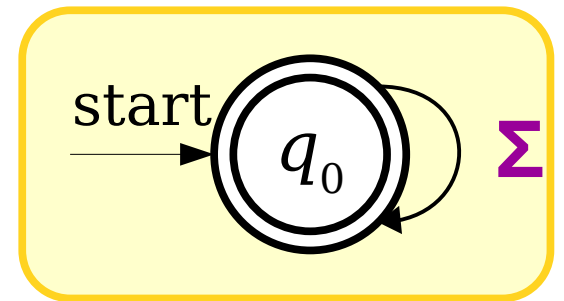
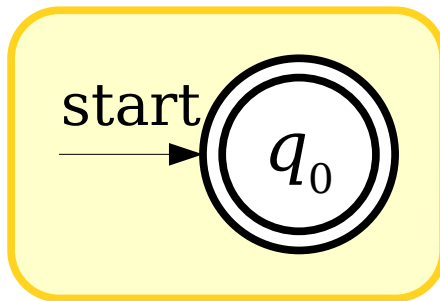
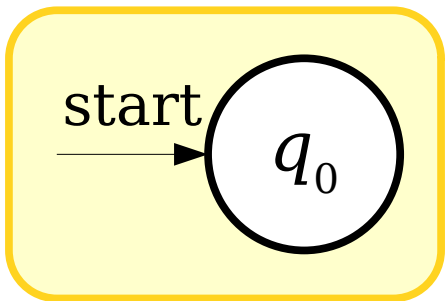
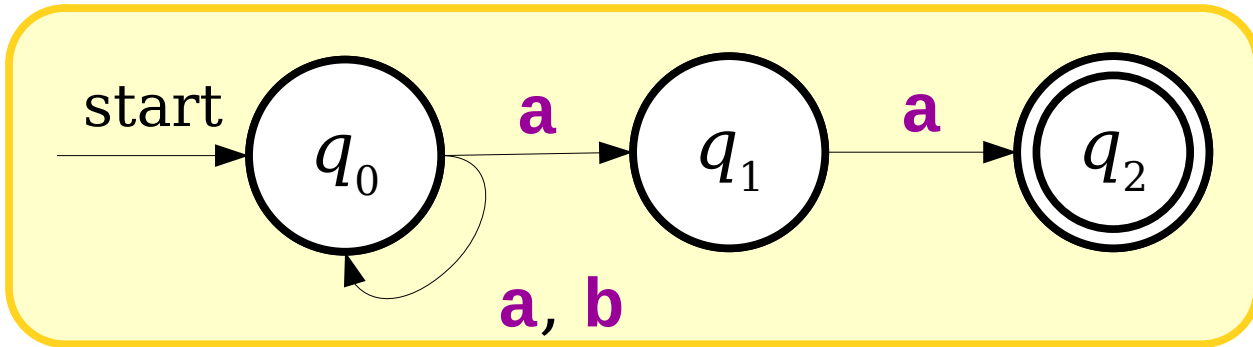
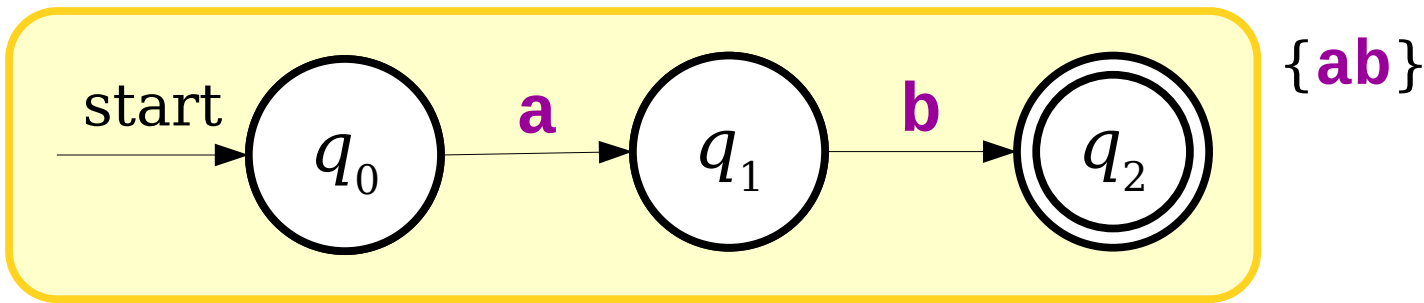


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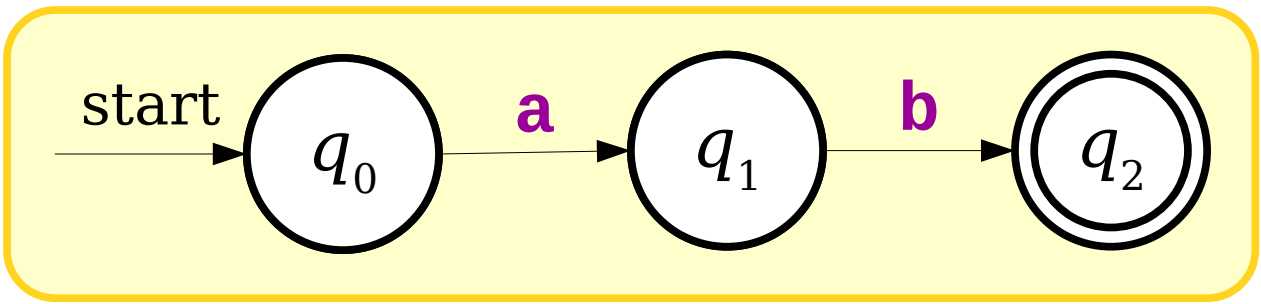
h	i	p
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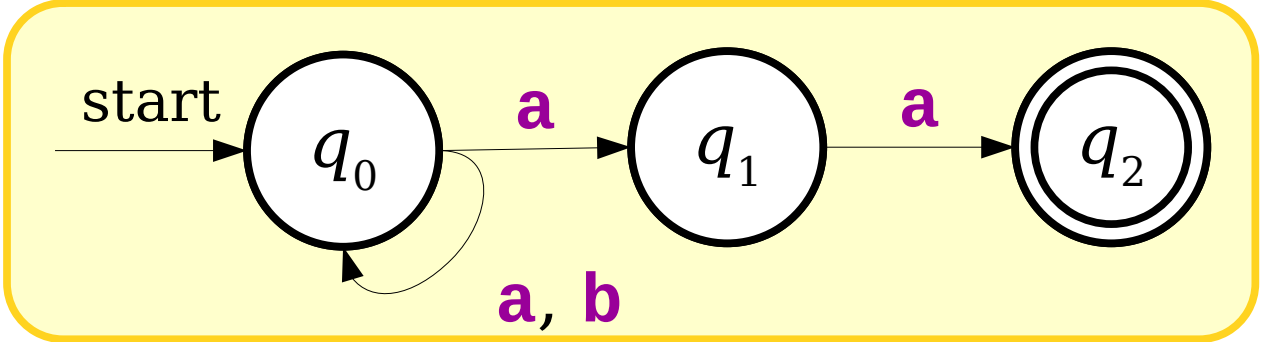
**Respond at**  
**[pollev.com/zhenglian740](http://pollev.com/zhenglian740)**

The **language of an NFA** is  
 $\mathcal{L}(N) = \{ w \in \Sigma^* \mid N \text{ accepts } w \}$ .  
 What is the language of each NFA?  
 (Assume  $\Sigma = \{a, b\}$ .)

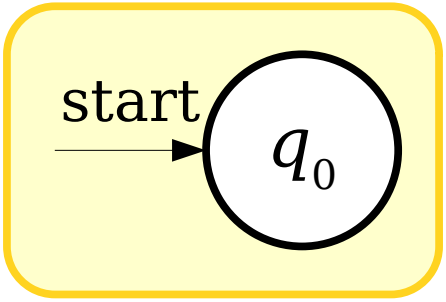


{ab}

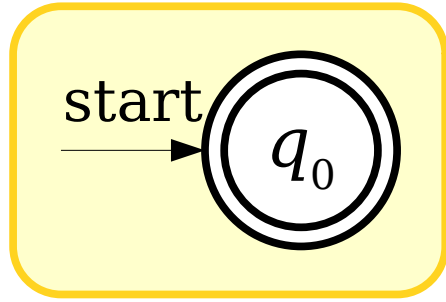
**Question to ponder:**  
 Why is the answer  
 $\{ w \in \Sigma^* \mid w \text{ ends in } \mathbf{aaa} \}$  not correct?



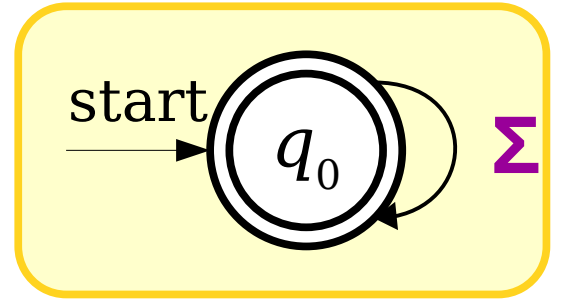
$\{ w \in \Sigma^* \mid w \text{ ends in } \mathbf{aa} \}$



$\emptyset$



{ $\epsilon$ }



$\Sigma^*$

Note that flipping the accept and reject states of an NFA doesn't always give an NFA for the complement of the original language. (Why?)

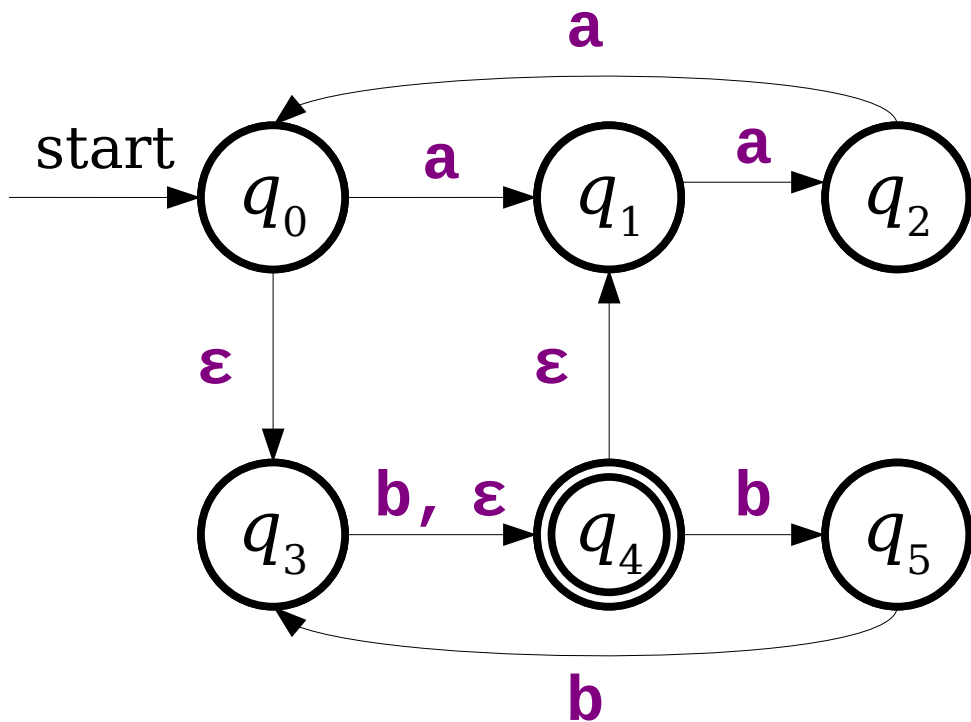
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# $\epsilon$ -Transitions

- NFAs have a special type of transition called the  **$\epsilon$ -transition**.
- An NFA may follow any number of  $\epsilon$ -transitions at any time without consuming any input.

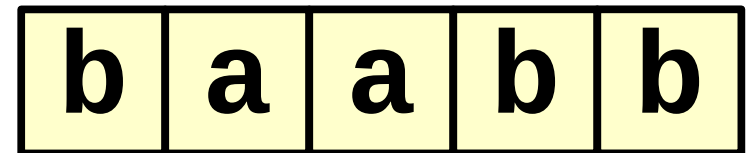
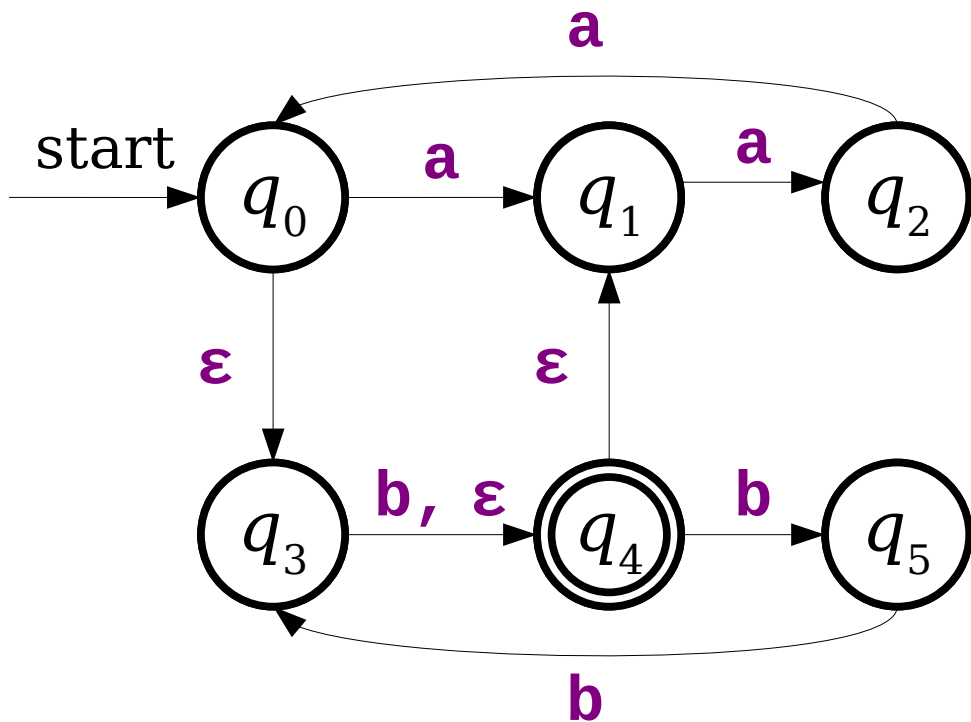
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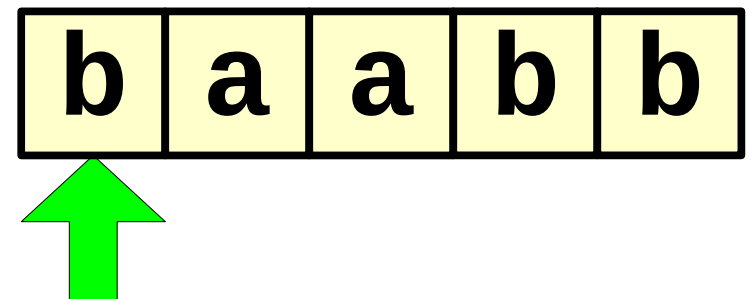
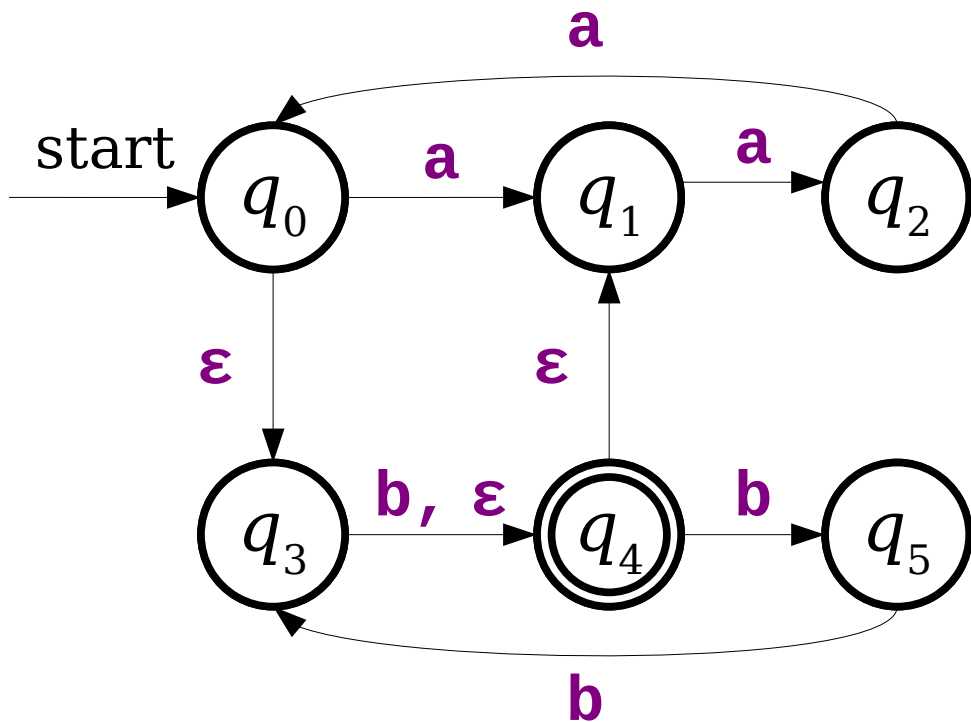
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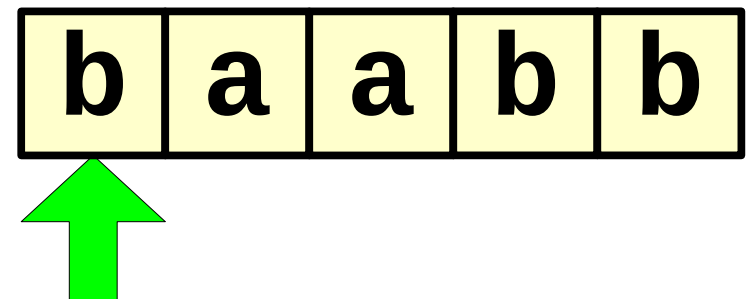
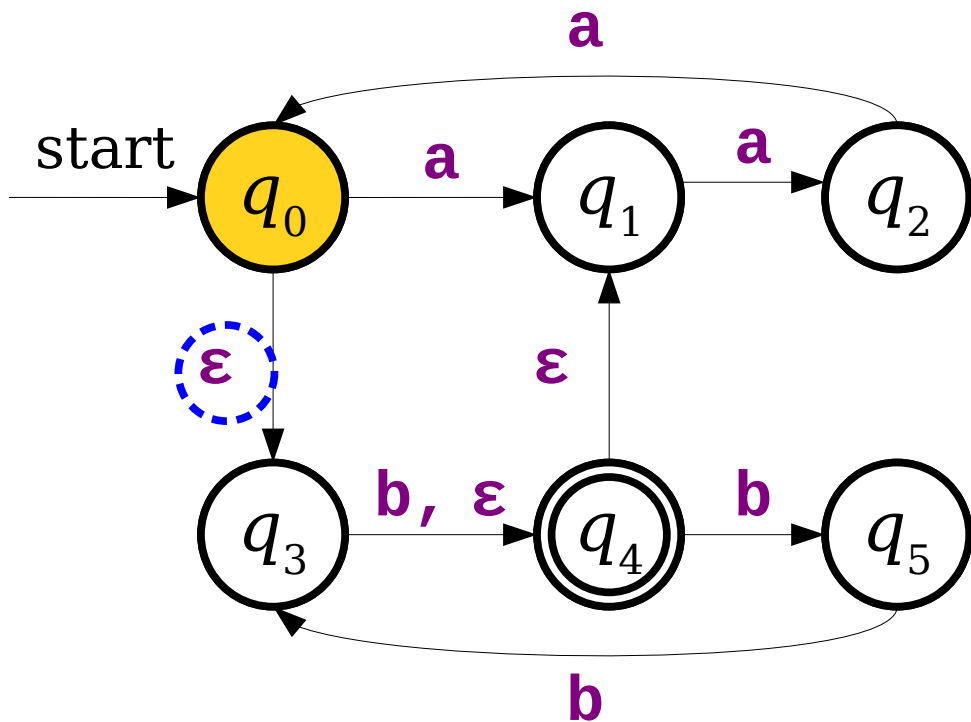
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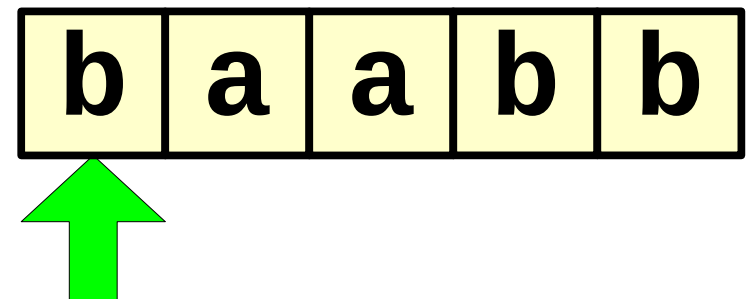
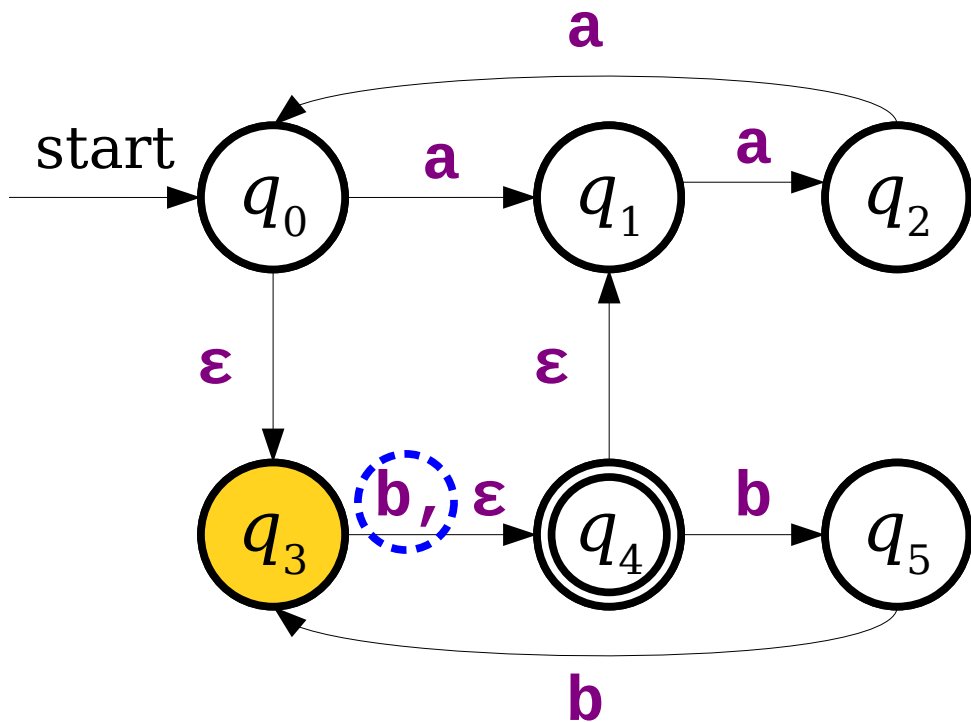
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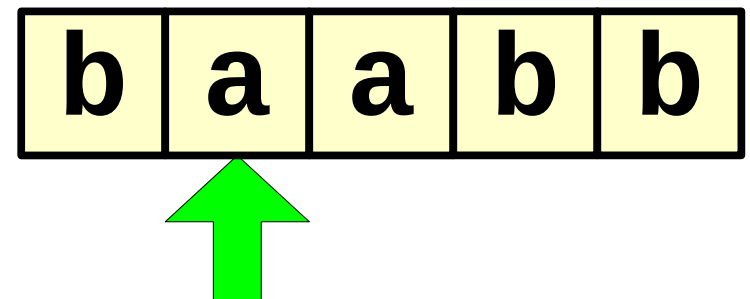
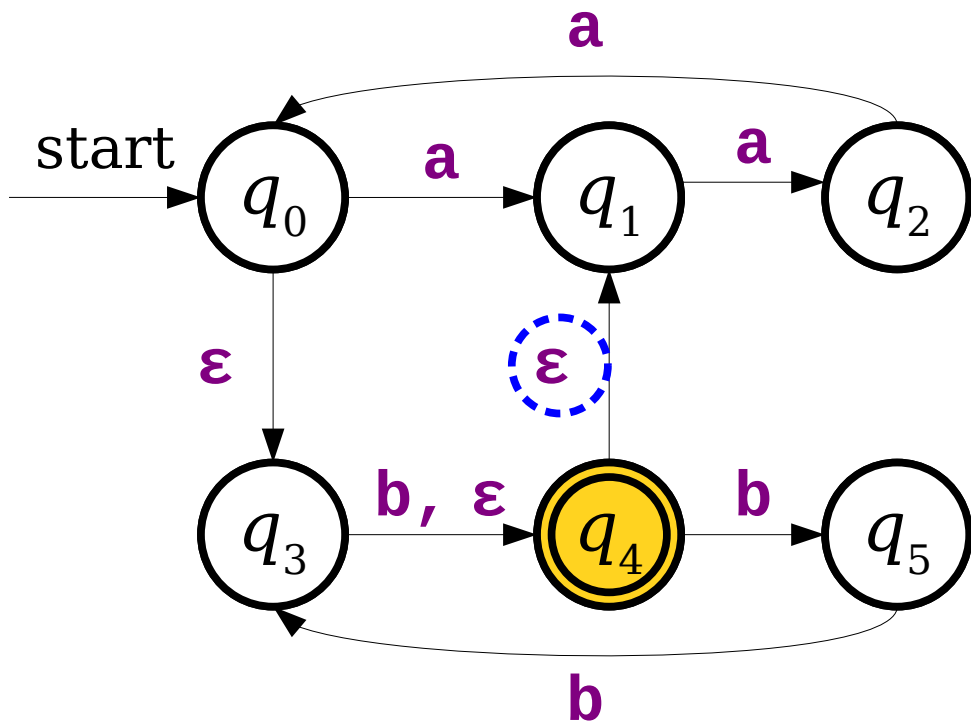
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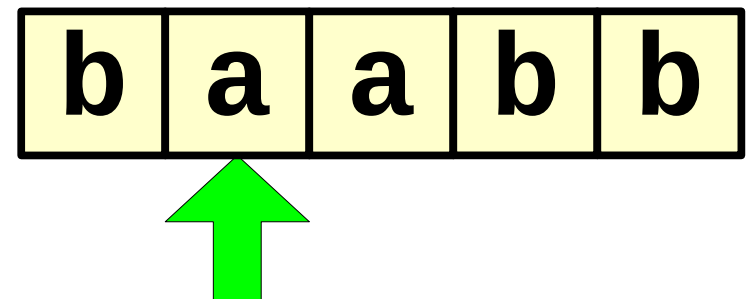
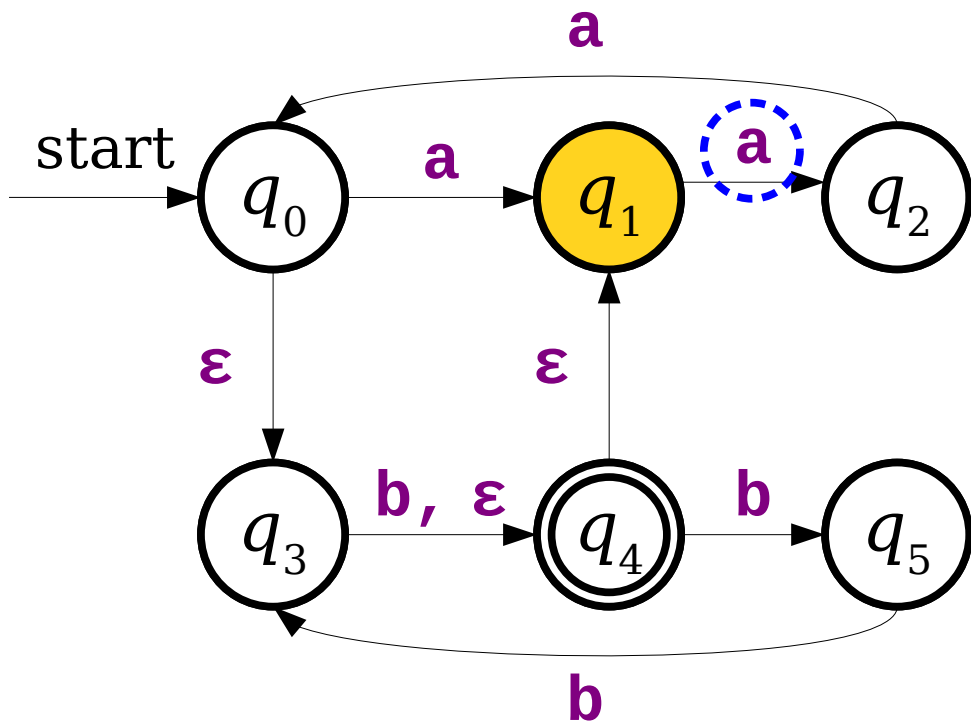
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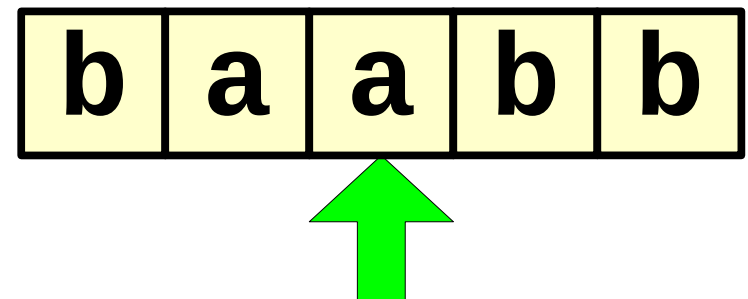
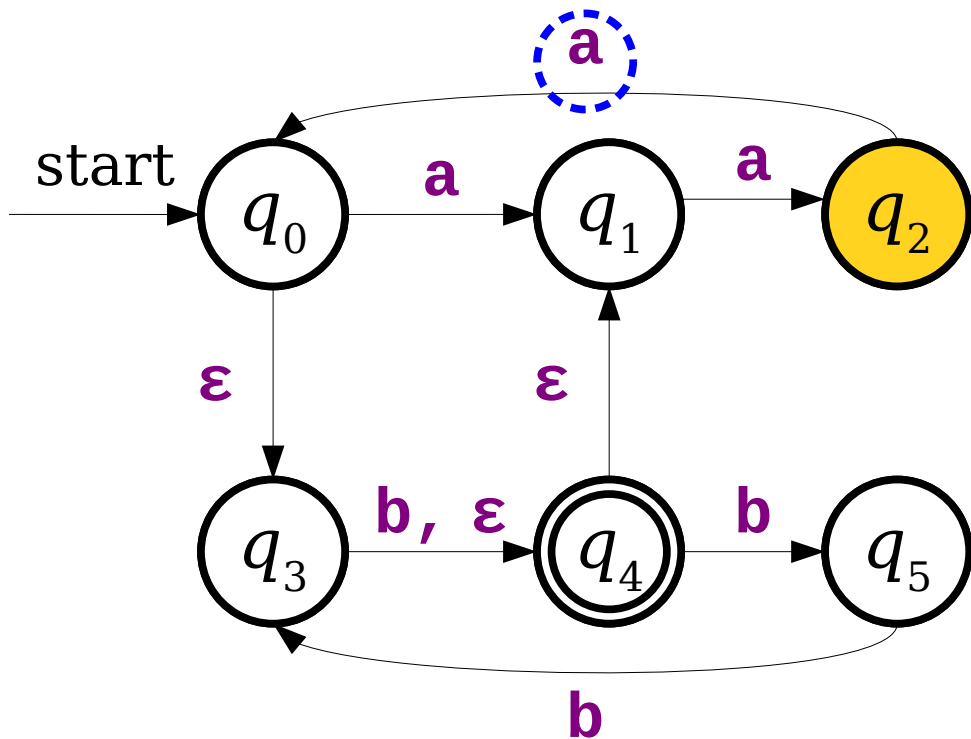
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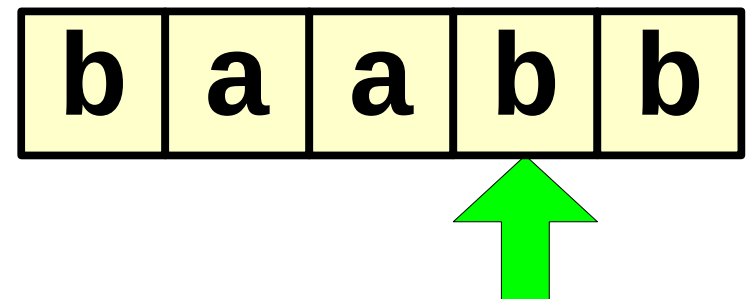
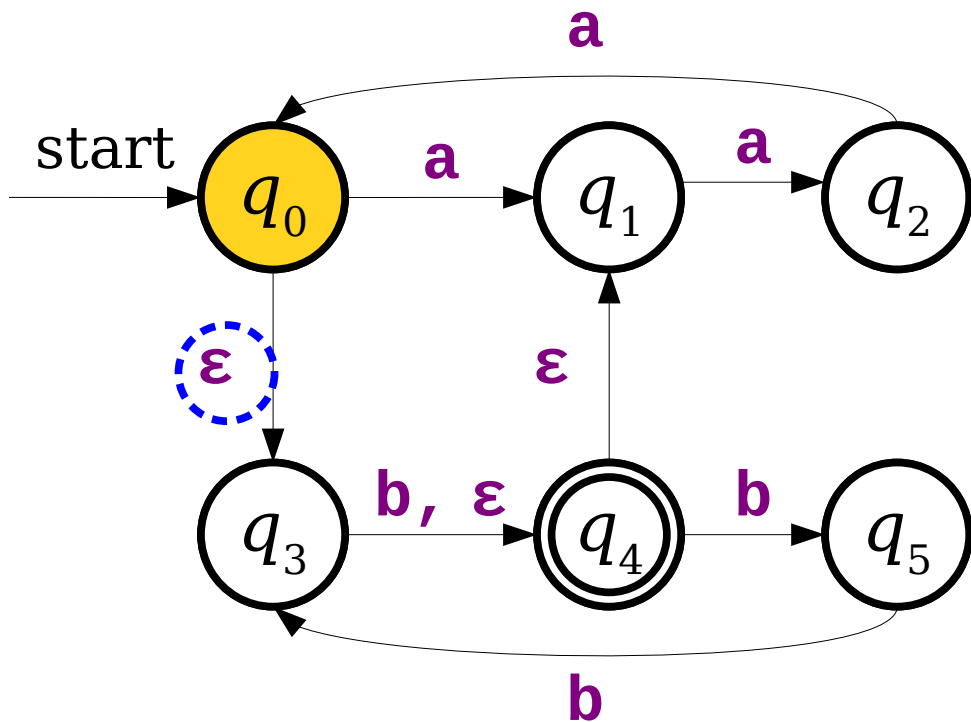
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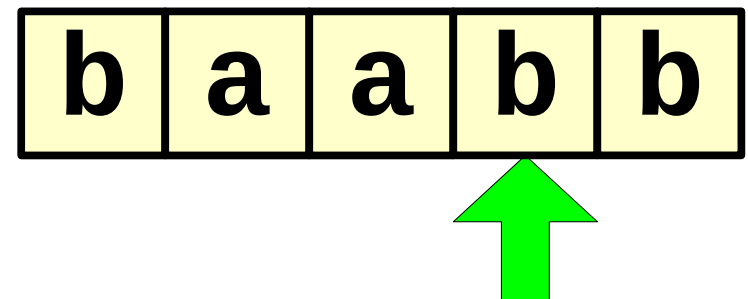
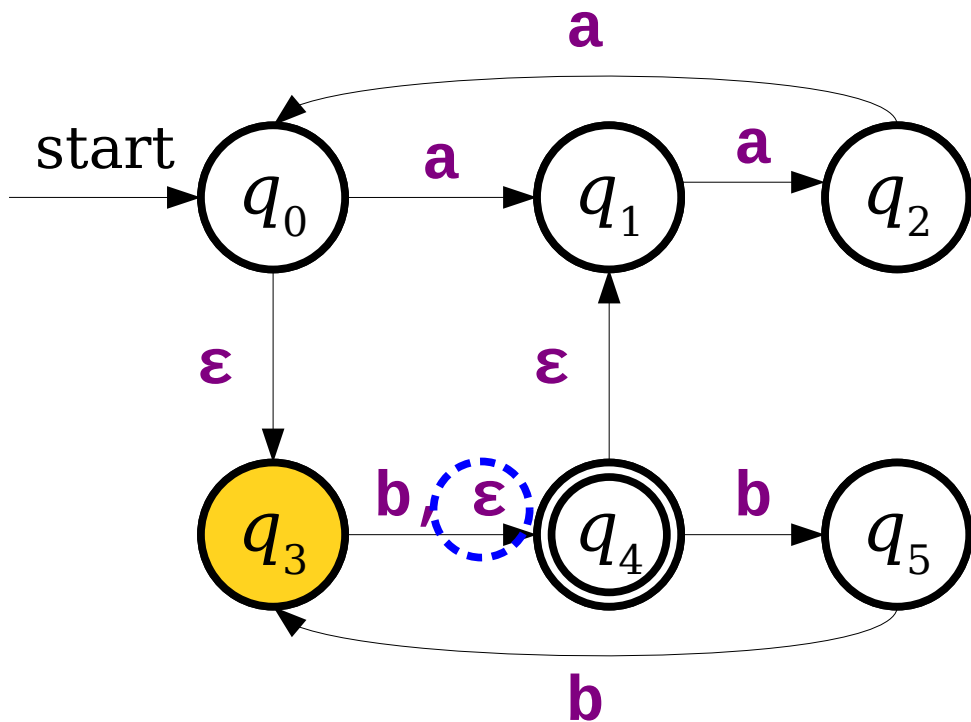
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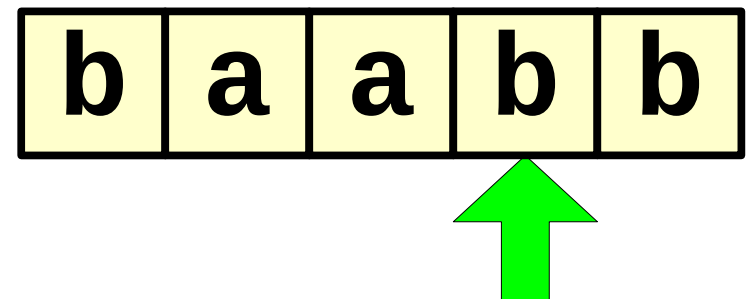
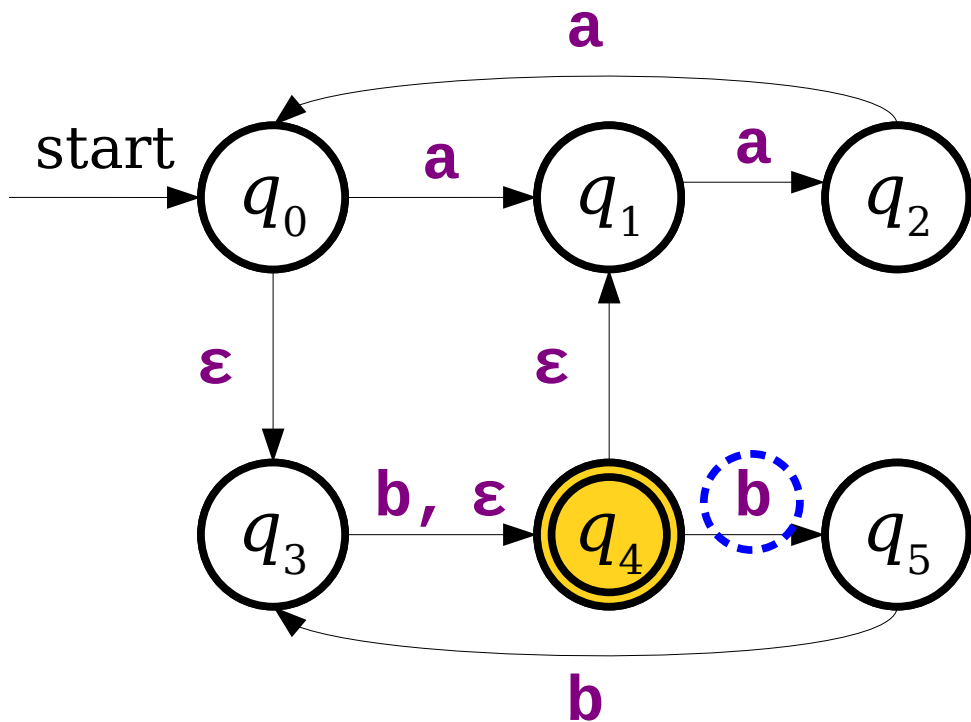
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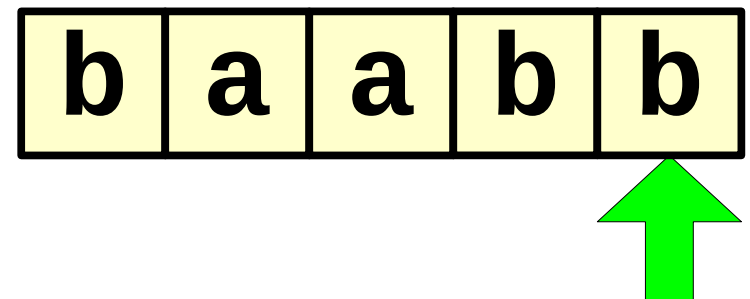
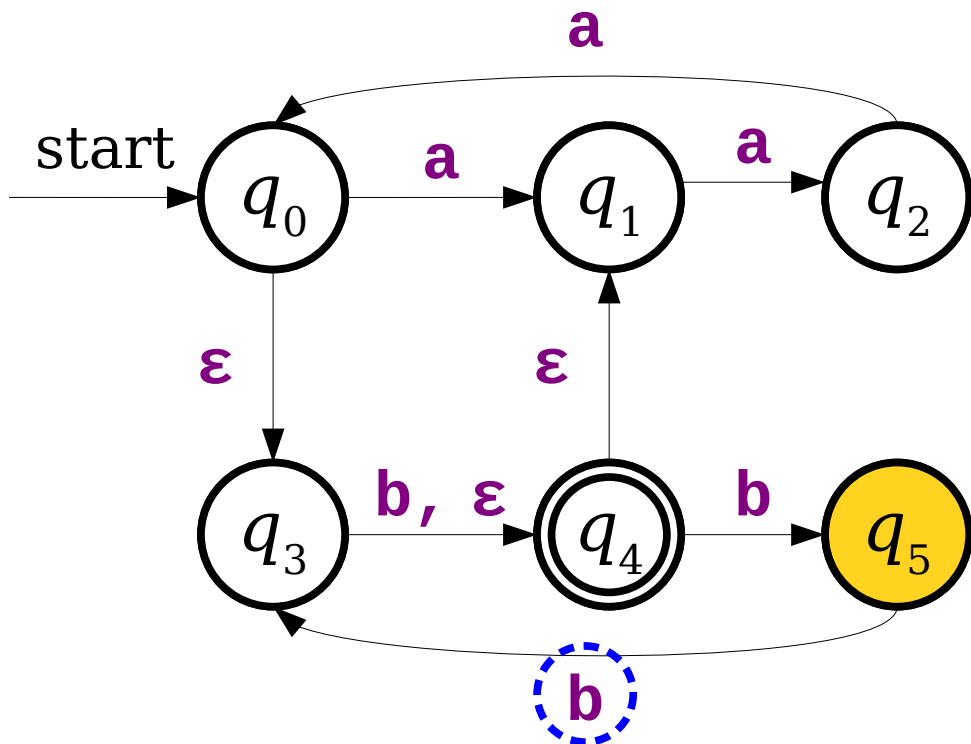
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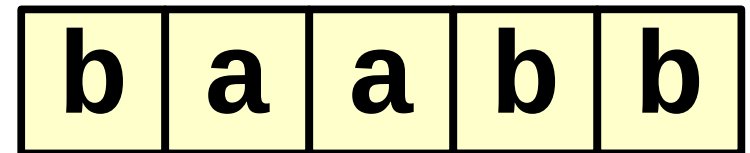
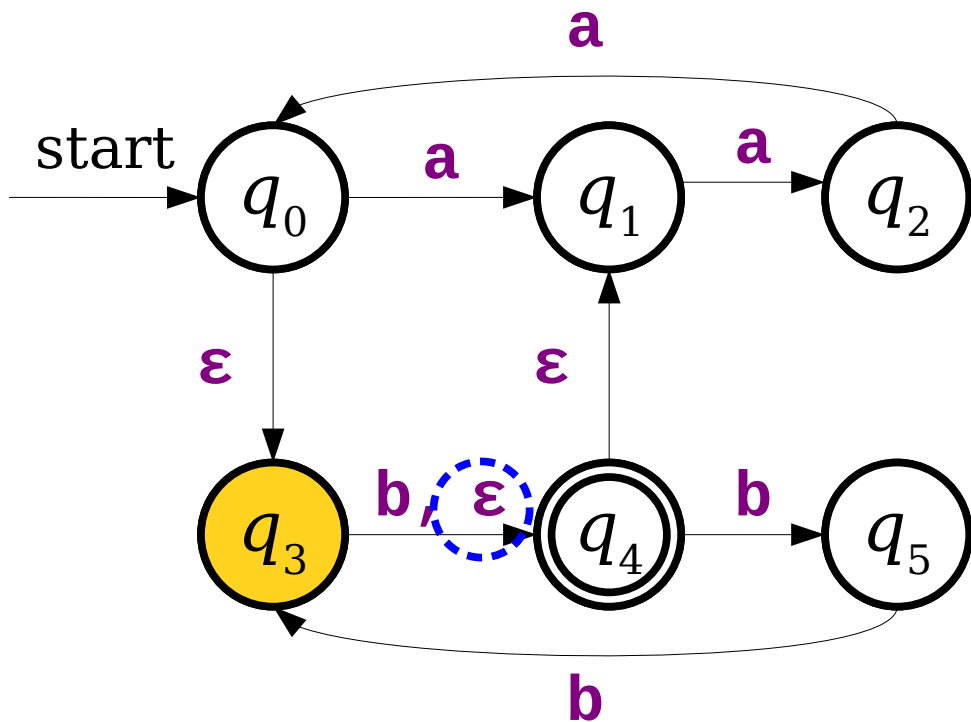
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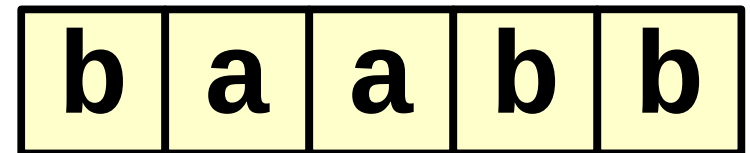
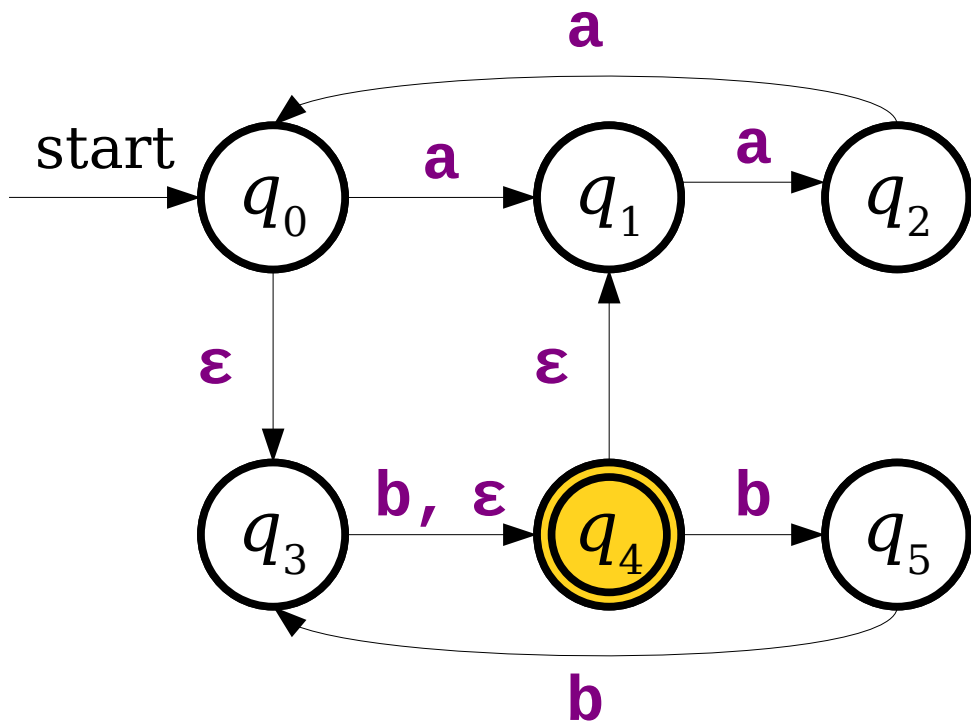
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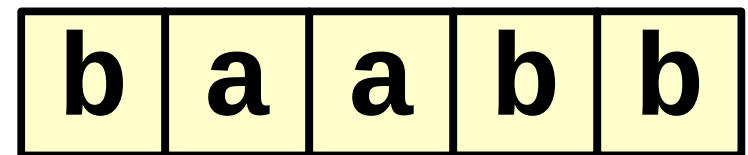
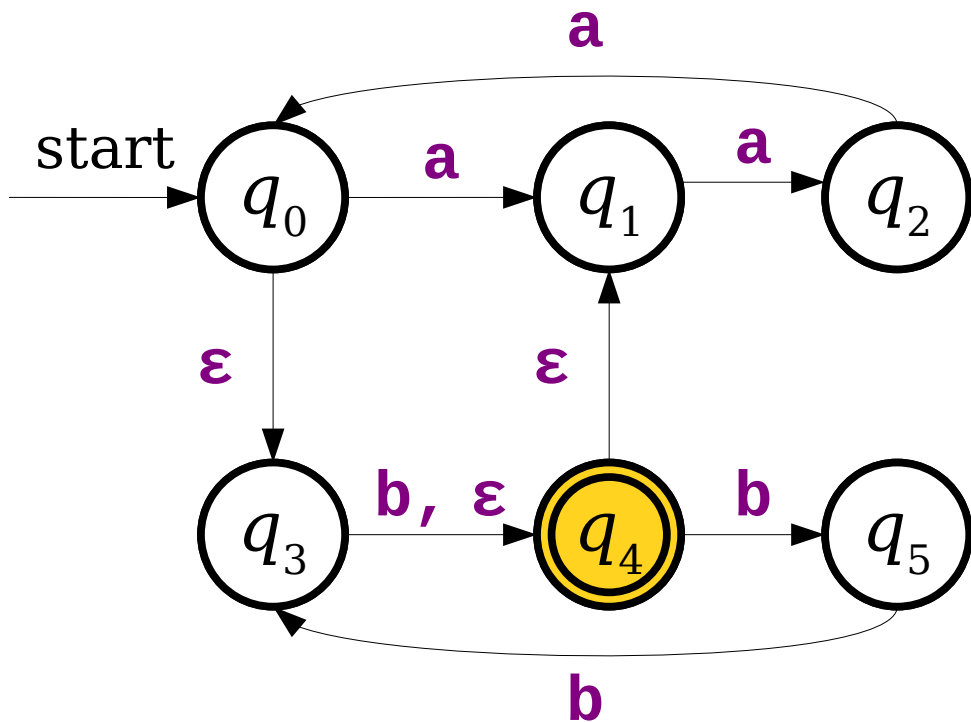
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- An NFA may follow any number of  $\epsilon$ -transitions at any time without consuming any input.
- NFAs are not *required* to follow  $\epsilon$ -transitions. It's simply another option at the machine's disposal.

NFAs feel like automata with magical superpowers. Just how powerful are they?

# Next Time

- ***The Powerset Construction***
  - So beautiful. So elegant. So cool!
- ***Closure Properties***
  - Can we combine regular languages?
- ***Language Transformations***
  - What's the deal with the notation  $\Sigma^*$ ?